# Statistical Pitfalls and Lessons from a Model of Human Decision-Making at Chess

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[1] Joint work with Tamal Tanu Biswas and with grateful acknowledgment to UB's Center for Computational Research (CCR)

## Chess History, Ancient and Modern

- Chess, either in Four Army form (Chatur-Angha) or today's White & Black, was known 2,500 years ago on the Subcontinent.
- Required knowledge for military commanders. Many conquests.
- Final conquest in 1997 by army of. . . processors. **Deep Blue**.
- Later conquered in 2017 by army of. . . nothing: **AlphaZero**.
- Now the army of handheld devices running chess programs (called engines) can defeat Carlsen, Anand, Kramnik, Kasparov, anyone.
- Since 2006, real and alleged chess cheating has been a major problem.
- First person caught and banned: Umakant Sharma, banned 12/2006 for 10 years by the AICF. Has a Wikipedia page,
- I advise the World Chess Federation (FIDE) on cases, "too many..."
- My statistical model has many other uses. My current CSE712 seminar may help to sharpen it.

## Elo Rating System

- Named for the Hungarian-American statistician **Arpad Elo**, the system gives every player $P$ a number $R_P$ representing skill.
- Defined by Logistic Curve: expected win % $p$ given by

$$p = \frac{1}{1 + \exp(c\Delta)}$$

where $\Delta = R_P - R_O$ is the difference to your opponent's rating and $c$ is a conversion constant.
- USCF takes $c = (\ln 10)/400$, so 200-pointse $\approx$ 75% expectation.
- **Class Units**: 2000–2200 = Expert, 2200–2400 = Master, 2400–2600 is typical of International/Senior Master and Grandmaster ranks, 2600–2800 = "Super GM,"; Carlsen 2857, 3 others over 2800, Anand 2770. Adult beginner $\approx$ 600, kids $\to$ 100.
- Komodo 11.1.3 **3414**?, Stockfish 9+ **3447**?, Houdini 6 **3410**?, Fire 6.1 **3298**... So computers $\approx$ "Class 14"—a kind of "Moore's Law."
- So **AlphaZero > 3500**? Higher than my measures of perfection...

## Reducing Chess to Numbers

- Chess engines all work by *incremental search* in rounds of increasing *depth* $d = 1, 2, 3, \ldots$
- For each round $d$ and legal move $m_i$ the program outputs a value $v_{i,d}$ in units of 0.01 called *centipawns*, figuratively 100ths of a pawn value (roughly P = 1, N = 3, B = 3+, R = 5, Q = 9).
- Values by Stockfish 6 in key Kramnik-Anand WC 2008 position:

| Move | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nd2 | 103 | 093 | 087 | 093 | 027 | 028 | 000 | 000 | 056 | -007 | 039 | 028 | 037 | 020 | 014 | 017 | 000 | 006 | 000 |
| Bxd7 | 048 | 034 | -033 | -033 | -013 | -042 | -039 | -050 | -025 | -010 | 001 | 000 | -009 | -027 | -018 | 000 | 000 | 000 | 000 |
| Qg8 | 114 | 114 | -037 | -037 | -014 | -014 | -022 | -068 | -008 | -056 | -042 | -004 | -032 | 000 | -014 | -025 | -045 | -045 | -050 |
| ... | | | ... | | | ... | | | ... | | | ... | | | ... | | | ... | |
| Nxd4 | -056 | -056 | -113 | -071 | -071 | -145 | -020 | -006 | 077 | 052 | 066 | 040 | 050 | 051 | -181 | -181 | -181 | -213 | -213 |

- Note that two moves have "equal-top value" (EV).
- This happens in 8–10% of positions.
- *These values are* (currently) *the only chess-specific inputs.*

# A Predictive Analytic Model

1. Domain: A set $T$ of decision-making situations $t$.
   Chess game turns
2. Inputs: Values $v_i$ for every option at turn $t$.
   Computer values of moves $m_i$
3. Parameters: $s, c, \dots$ denoting skills and levels.
   Trained correspondence $P(s, c, \dots) \longleftrightarrow$ Elo rating $E$
4. Main Output: Probabilities $p_i$ $(= p_{t,i})$ for $P(s, c, \dots)$ to select option $i$ (at turn $t$).
5. Derived Outputs:
   - **MM%, EV%, AE** and other aggregate statistics.
   - Projected confidence intervals for them—via Multinomial Bernoulli Trials plus an adjustment for correlation between consecutive turns.
   - **Intrinsic Performance Ratings** (IPRs) for the players.

## How the Model Operates

- Given $s, c, \ldots$ and each legal move $m_i$ with value $v_i$ (at top depth), the model computes a *proxy value*

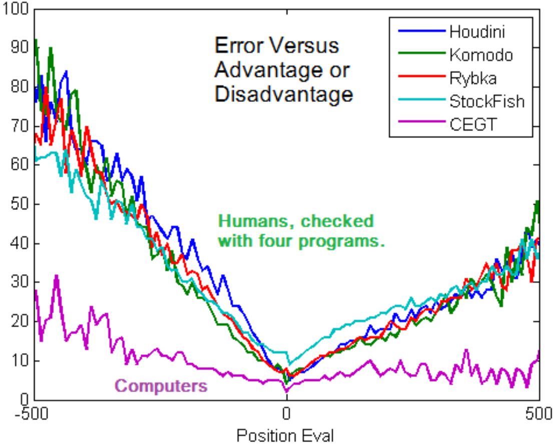
$$u_i = g_{s,c}(\delta(v_1, v_i)),$$

  where $\delta(v_1, v_i)$ scales down the raw difference $v_1 - v_i$ in relation to the overall position value $v_1$, and $g = g_{s,c}$ is a family of curves giving $g(0) = 1$, $g(z) \to 0$.
- Intuitively, $1 - u_i$ is the "perceived inferiority" of the move $m_i$.
- Besides $g$, the model picks a function $h(p_i)$ on probabilities.
- Could be $h(p) = p$ (bad), log (good enough?), $H(p_i)$, *logit*...
- The **Original Main Equation:**

$$\frac{h(p_i)}{h(p_1)} = u_i = \exp(-\left(\frac{\delta(v_1, v_i)}{s}\right)^c).$$

- *Any such value-based model entails* $v_1 = v_2 \implies p_1 = p_2$.

# Why the Scaling?




Scaling $\delta(u, v) = \int_{x=u}^{x=v} \frac{1}{1+Cx}\, dx$ (for $x > 0$) levels out differences.

# Five Expectations—and Curveballs/Googlies:

1. Equal values yield equal behavior.
2. Unbiased data-gathering yields unbiased data.
3. Biases that are obvious will show up in the data.
4. If $Y$ is a continuous function of $X$, then a small change in $X$ produces a small change in $Y$.
5. Factors whose insignificance you demonstrated will stay insignificant when you have 10x–100x data.
6. *OK, 1.5:* Secondary aspects of standard library routines called by your data-gathering engines won't disturb the above expectations.

**Googlies**: *Data points have histories*, *notionally* unbiased/ continuous/... need not imply *factually* unbiased/ continuous/..., and *zero-sigma* results can be artifacts too.

## $X$ and $Y$ and $Z$

- $X$ = *values of chess moves.*
- $Y$ = *performance indicators* of (human) players:
    - **MM%** = how often the player chose the move listed first by the engine in value order.
    - **EV%** = how often the player chose the first move or one of equal value, as happens in 8–10% of positions.
    - **ASD** = the average scaled difference in value between the player's chosen move $m_i$ and the engine's first move $m_1$.
- $Z$ = Elo rating
- The 2-parameter model is fitted simply by setting the projected MM% and ASD equal to the sample means.
- Resulting EV estimator is biased "conservatively" (against false positives).

# The Data: Old and New

- **Old:** Over 6 million moves of **Multi-PV** data: $>$ 500 GB.
- Over 120 million moves of **Single-PV** data: $>$ 200 GB
- $=$ 350 million pages of text data at 2k/page.
- All taken on two quad-core home-style PC's plus a laptop using the GUI. This involved **retaining hashed move values** between game turns—which is the normal playing mode and only GUI option.
- **New—using CCR:** Every published high-level game since 2014 in **Single-PV** mode.
- **Master training sets** of 1.15 million moves by players of Elo ratings 1025, 1050, 1075, 1100, . . . (stepping by 25) . . . , 2750, 2775, 2800, all in **Multi-PV mode**.
- Taken with Komodo 10 and Stockfish 7, *all years since 1971*.

# First Googly: An "ESP Test"

- In 8%–10% of positions, engine gives the top two moves the same value.
- Even more often, *some* pair of moves in the top 10 (say) will end up tied. Conditioned on one of them having been played, let us invite humans to guess **which move is listed first by the program**.
- The values are identical to the engine: it would not matter to the quality of the output which one the engine listed first. The values give no human reason to prefer one over the other.
- So this is a kind of ESP test. *How well do humans perform on it?*
- PEAR—Princeton Engineering Anomalies Research—notorious ESP project.
- PEAR did 10,000s–100,000s of trials, trying to judge significance of deviations like 50.1% or even 50.01%.
- How about *my* ESP test??

# Sensitivity—Plotting $Y$ against $X$

Conditioned on one of the top two moves being played, if their values (old: Rybka 3, depth 13; new: Stockfish and Komodo, depths 19+) differ by...:

1. **0.01**, the higher move is played 53–55% of the time.
2. **0.02**, the higher move is played 58–59% of the time.
3. **0.03**, the higher move is played 60–61% of the time.
4. **0.00**, the higher move is played 55–59% of the time.

- Last is not a typo—see post "**When is a Law Natural?**"
- Similar 58%-42% split seen for any pair of tied moves, all Elo over 2000, down to 55%–45% for Elo 1050. What can explain it?
- Relation to slime molds and other "semi-Brownian" systems?

# History and "Swing" over Increasing Depths

The ____ of drug-resistant strains of bacteria and viruses has ____ researchers' hopes that permanent victories against many diseases have been achieved.

(a) vigor . . corroborated

(b) feebleness . . dashed

(c) proliferation . . blighted

(d) destruction . . disputed

(e) disappearance . . frustrated

(source: itunes.apple.com)

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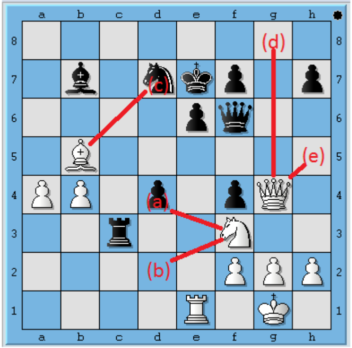


| Move | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
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# Measuring "Swing" and Complexity and Difficulty

- Non-Parapsychological Explanation: *Stable* Library Sorting.
- Chess engines sort moves from last depth to schedule next round of search.
- Stable $\rightarrow$ lower move jumps to 1st only with *strictly higher* value.
- Lead moves tend to have been higher at lower depths. Lower move "swings up."
- Formulate numerical measure of swing "up" and "down" (a trap).
- When best move swings up **4.0–5.0** versus **0.0–1.0**, players rated 2700+ find it only **30%** versus **70%**.
- **Huge differences** $\Longrightarrow$ corrections to the **main equation**.
- Will also separate *performance* and *prediction* in the model.

## The New Model—as of today!

- My old idea was to extend the main equation to a weighted linear combinationover depths governed by a "peak depth" parameter $d$:

$$\frac{h(p_i)}{h(p_1)} = 1 - x_i = u_i = \sum_{j=1}^{D} w_j \exp(-\left(\frac{\delta(v_{1,j}, v_{i,j})}{s}\right)^c),$$

- Led to horrible fitting landscape, many local minima...
- Simpler idea advocated by my student Tamal Biswas: first define some concrete measure of the "swing" of move $m_i$, *viz.*

$$sw(i) = \frac{1}{D}\sum_{j=1}^{D}(\delta_{i,j} - \delta_{i,D}).$$

- Then introduce a new parameter $h$ (for nautical "heave") and fit:

$$\frac{h(p_i)}{h(p_1)} = 1 - x_i = \exp(-\left(\frac{\delta(v_1, v_i) + h \cdot sw(i)}{s}\right)^c).$$

## How the Model is Fitted

- Given $s, c, h$, compute proxy values $u_i = g_{s,c,h}(v_1, v_i)$.
- Solve for $p_1, \ldots, p_i, \ldots$ subject to $\sum_i p_i = 1$ such that

$$\frac{h(p_i)}{h(p_1)} = u_i; \qquad \text{specific choice:} \qquad \frac{\log(1/p_1)}{\log(1/p_i)} = u_i.$$

- This gives $P_{s,c,h} : p_i = p_1^{1/u_i}$ for each $i$.
- No closed form? Hence inner regression to find $\{p_i\}$ that we will *memoize*.
- Outer regression applies $P_{s,c,h}$ to generate projected MM%, EV%, ASD.
- Regress over $s, c, h$ to fit to sample means. Expensive!
- But appears to work well: the 2nd-best, 3rd-best, 4th-best move frequencies fall into place all down the line.
- Another "natural law"? At least indicates model is basically right...

# Second Googly

- **Single-PV** = normal playing (and cheating?) mode.
- **Multi-PV** values needed for main model equation.
- Does difference matter for **MM%, EV%, ASD**?
- *Value* of first move seems unaffected. However (plotting $Y$ vs. $Z$):

> Human players of all rating levels have 2–3% higher MM% and EV% to the Single-PV mode.

Thus my model is a biased predictor of MM% in Single-PV mode. Bias avoided by conducting test entirely in Multi-PV mode (arguably conservative). Why might this happen?

> Single-PV mode maximally retards "late-blooming" moves from jumping ahead in the stable sort.

## Third Googly: No Such Thing As Being "In Form"?

- I routinely "screen" 5,000+ games per week in SinglePV mode.
- Not my full model, just a simple "Raw Outlier Index" (ROI) from each player's MM%, ASD, and rating.
- Large "Open" tournaments have hundreds of players in a "Swiss System" (not knockout) format.
- The top 10-20 or so games are on auto-recording boards that can broadcast moves.
- Some tournament staffs type up the rest of the games from scoresheets submitted by players.
- Others do not—those tournaments I mark with `Avail` in filenames.
- After Round 1, the top boards have people who have done well in recent rounds.
- Hence `Avail` files skew massively toward "in form" players.
- But *no significant difference* in ROI (if anything, the opposite).
- No "Hot Hand" in chess? Or maybe nerves offset form?...

# Fourth Googly: A “Firewall at Zero

Surely $Y$ = the frequency of large errors (“blunders”) ought to be continuous as a function of $X$ = the value of the position. But:

| Elo 2600–2850 | Komodo 9.3 | | | | Stockfish 7 (modified) | | | |
|---|---|---|---|---|---|---|---|---|
| Value range | #pos | d10 | d15 | d20 | #pos | d10 | d15 | d20 |
| -0.30 to -0.21 | 4,710 | 9 | 13 | 18 | 4,193 | 13 | 10 | 14 |
| -0.20 to -0.11 | 5,048 | 11 | 10 | 13 | 5,177 | 6 | 9 | 11 |
| -0.20 to -0.01 | 4,677 | 11 | 13 | 16 | 5,552 | 8 | 9 | 16 |
| 0.00 exactly | 9,168 | 24 | 25 | 28 | 9,643 | 43 | 40 | 38 |
| +0.01 to +0.10 | 4,283 | 6 | 1 | 2 | 5,705 | 8 | 3 | 2 |
| +0.11 to +0.20 | 5,198 | 7 | 5 | 3 | 5,495 | 10 | 5 | 3 |
| +0.21 to +0.30 | 5,200 | 7 | 2 | 1 | 4,506 | 3 | 4 | 2 |

Reason evidently that 0.00 is a big *basin of attraction* in complex positions that may force one side to give perpetual check or force repetitions to avoid losing. Safety net provided $v_1 > 0$ but absent when $v_1 < 0$. Failure to charge adequately for large “notional errors.”

# Fifth Googly—Clearing Hash Does Matter

- Retaining hash apparently also retards "later-blooming" moves.
- Effect only 0.25–0.35%, not 2–3%, but significant now.
- Clearing is better for scientific reproducibility but further from actual playing conditions.

Thus my original "simple and self-evident" model needs substantial adjustment for all of these factors—to say nothing of factors like the scaling which I caught at the beginning. . .

To conclude on a philosophic note: "Big Data" is critiqued for abandoning *theory*. Need not be so—my chess model is theory-driven and "severely underfitted." *But theory cannot abandon data*—nor a full understanding of the *history* and *hidden biases* it may embody.

## A Sixth Lesson: Weighting and Bootstrap

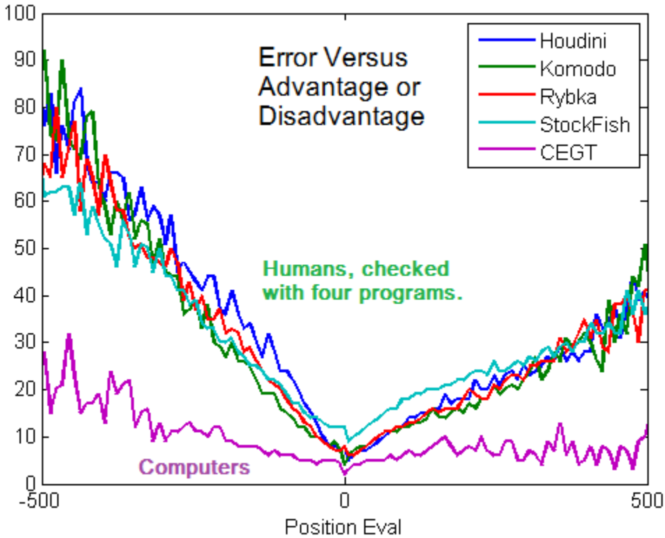
- This does not involve my model, only chess program evaluation functions $v = v(p)$ of positions $p$.
- Graph $v$ versus scoring frequency $e(v)$ from positions of value $v$.
- Fantastic logistic fit $e(v) = A + \frac{1-2A}{1+\exp(-Bv)}$, $B$ depends on rating.
- Has $R^2 > 0.9999999$ but what are the error bars on $B$?
- Can weight regression by number $N_v$ of positions of value $v$. Concentrated near $v = 0$.
- But cross-check by **Bootstrap** of $B$ is off by factor of 2.
- Instead of "$X$-side" weighting, can use $1/\sigma$ of "$Y$-side" instead.
- Not $\sim \sqrt{N_v}/2$ but rather $\sim \sqrt{e(v)(1-e(v))N_v}$. Different in tails.
- Eliminates the discrepancy from bootstrap results.

# Seventh Seal: Cross-Validation and Fitting Horror

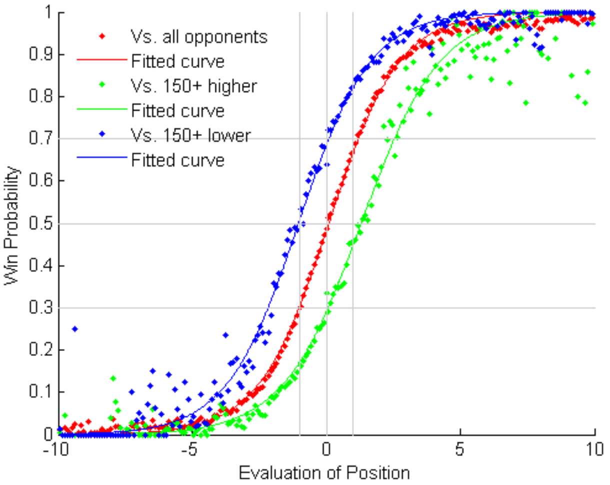
- The fitting of $s, c, h$ can be done in many other ways. . . .
- The model is "severely underfitted"—theory-heavy.
- How well does your favorite fitting method work?
- Maximum Likelihood Estimation: minimize $\sum_t \log(1/p_{t,i_t})$ where $i_t$ is the index of the played move at each game turn $t$.
- Performs relatively poorly—a known phenomenon with underfitting.
- In the 3- and 4-parameter models, *chaos breaks loose.* **Literally.**
- Segue to posts on the *Gödel's Lost Letter* blog:

"Unskewing the Election"
"Stopped Watches and Data Analytics"

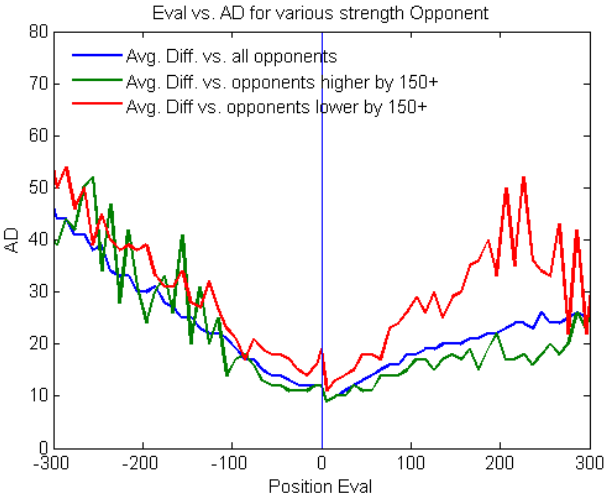
# Extras: Human Versus Computer Phenomena



# Human Versus Computer Phenomena



# Eval-Error Curve With Unequal Players



# Computer and Freestyle IPRs

Analyzed Ratings of Computer Engine Grand Tournament (on commodity PCs) and PAL/CSS Freestyle in 2007–08, plus the Thoresen Chess Engines Competition (16-core) Nov–Dec. 2013.

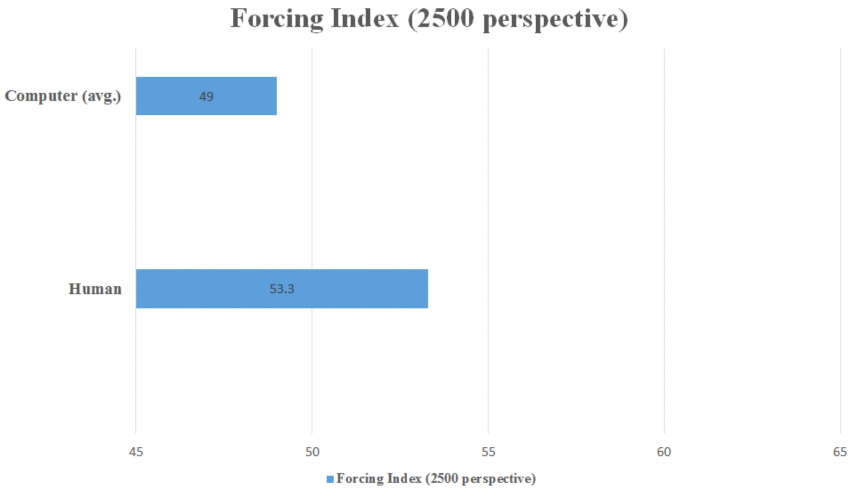
| Event | Rating | $2\sigma$ range | #gm | #moves |
|---|---|---|---|---|
| CEGT g1,50 | 3009 | 2962–3056 | 42 | 4,212 |
| CEGT g25,26 | 2963 | 2921–3006 | 42 | 5,277 |
| PAL/CSS 5ch | 3102 | 3051–3153 | 45 | 3,352 |
| PAL/CSS 6ch | 3086 | 3038–3134 | 45 | 3,065 |
| PAL/CSS 8ch | 3128 | 3083–3174 | 39 | 3,057 |
| TCEC 2013 | 3083 | 3062–3105 | 90 | 11,024 |

# Computer and Freestyle IPRs—To Move 60

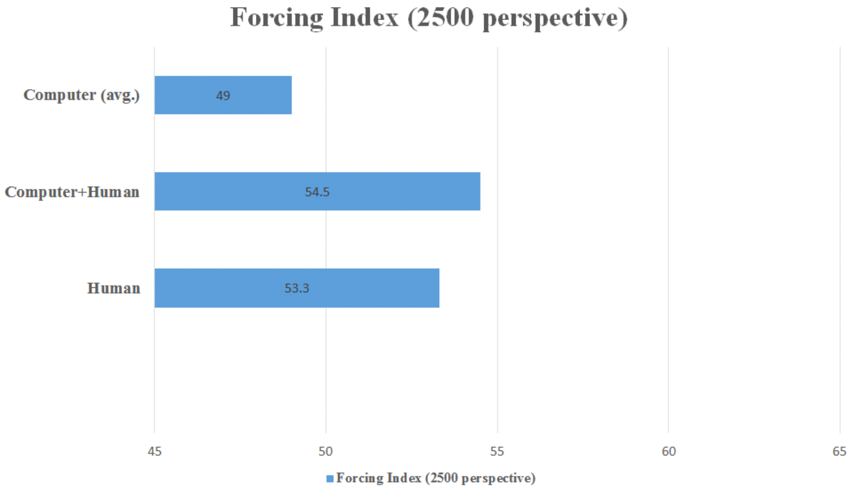
Computer games can go very long in dead drawn positions. TCEC uses a cutoff but CEGT did not. Human-led games tend to climax (well) before Move 60. This comparison halves the difference to CEGT, otherwise similar:

| Sample set | Rating | $2\sigma$ range | #gm | #moves |
|---|---|---|---|---|
| CEGT all | 2985 | 2954–3016 | 84 | 9,489 |
| PAL/CSS all | 3106 | 3078–3133 | 129 | 9,474 |
| TCEC 2013 | 3083 | 3062–3105 | 90 | 11,024 |
| CEGT to60 | 3056 | 3023–3088 | 84 | 7,010 |
| PAL/CSS to60 | 3112 | 3084–3141 | 129 | 8,744 |
| TCEC to60 | 3096 | 3072–3120 | 90 | 8,184 |

# Degrees of Forcing Play

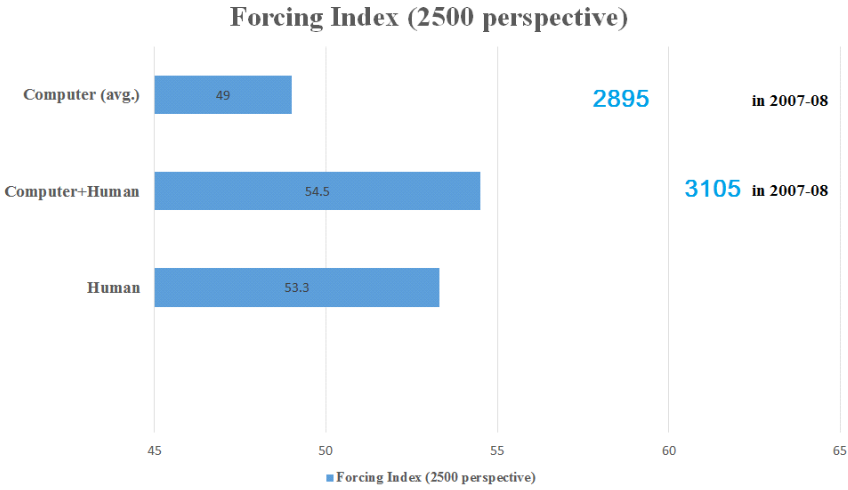


# Add Human-Computer Tandems



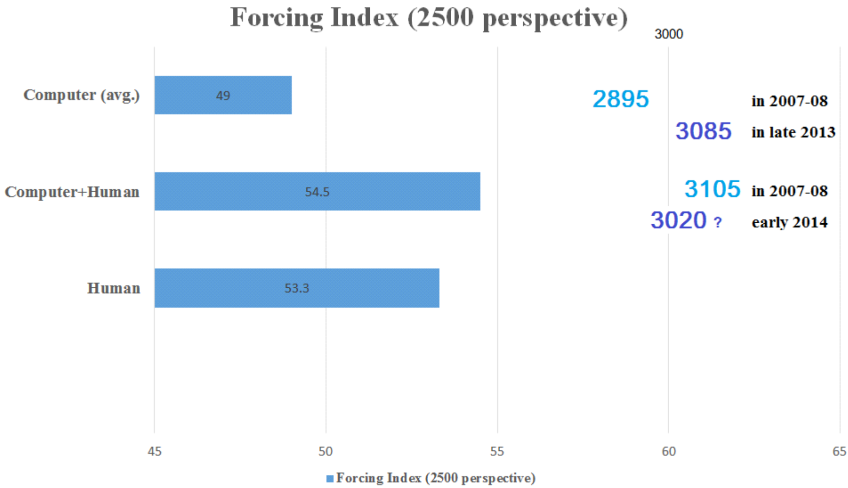

Evidently the humans called the shots. But how did they play?

# 2007–08 Freestyle Performance




Adding 210 Elo was significant. Forcing but good teamwork.

# 2014 Freestyle Tournament Performance




**Tandems had marginally better W-L, but quality not clear...**