Statistical Chess Cheating Detection

Marshall Chess Club, with USCF and CFC

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Two-Stage Setup

- Aims to empower tournament officials with positive as well as negative feedback and avoid rush to judgment.
- In two stages: **Screening** and **Full Test**.
- Screening does not make formal statistical judgments.
- Makes a simple "box score" of agreements to the chess engine being tested and the **scaled** average centipawn loss from disagreements.
- Creates a **Raw Outlier Index (ROI)** on the same 0-100 scale as flipping a fair coin 100 times.
- Here 50 is the expectation *given one's rating* and 5 is the standard deviation, so the "two-sigma normal range" is 40-to-60.
- Like medical stats except **indexed** to common **normal** scale.
- 65 = amber alert, 70 = code orange, 75 = red. Example.
- **Completely data driven.** Rapid and Blitz trained on **in-person** events in 2019. Works for "non-cheating middle" in online play.

The Full Test—A Predictive Analytic Model

Means that the model:

- Addresses a series of events or decisions,each with possible outcomes $m_1, m_2, \ldots, m_j, \ldots$
- Assigns to each m_j a probability p_j.
- Projects risk/reward quantities associated to the outcomes.
- Also assigns *confidence intervals* for p_j and those quantities.

Mine is based on a **utility function / loss function** in a standard way except for being log-log linear, not log-linear. Has parameters

- s for "sensitivity"—strategic judgment.
- c for "consistency" in surviving tactical minefields.
- h for "heave" or "Nudge"—obverse to depth of thinking.

Trained on all available in-person classical games in 2010–2019 between players within 10 Elo of a marker 1025, 1050, . . . , 275, 2800, 2825. Wider selection below 1500 and above 2500.

How it Works

- Take s, c, h from a player's rating (or "profile").
- Generate probability p_i for each legal move m_i.
- Paint m_i on a 1,000-sided die, 1,000p_i times.
- **Roll the die**.
- (Correct after-the-fact for chess decisions not being independent.)

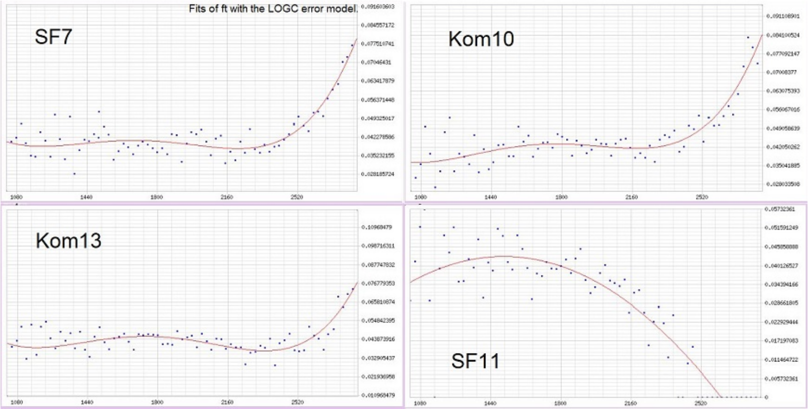
The statistical application then follows by math known since the 1700s. (Example of "Explainable AI" at small cost in power.)

Validate the model on millions of randomized trials involving "Frankenstein Players" to ensure conformance to the standard bell curve at all rating levels.

See: Published papers and articles on Richard J. Lipton's blog **Gödel's Lost Letter and P=NP** which I partner.

How Well Does It Work?

Internal evidence that it gives $(1+\epsilon)$ relative error with $\epsilon \approx 0.04$ for most rating levels. Means it supports betting on chess moves with only 5% "vig" needed to avoid *arbitrage*.



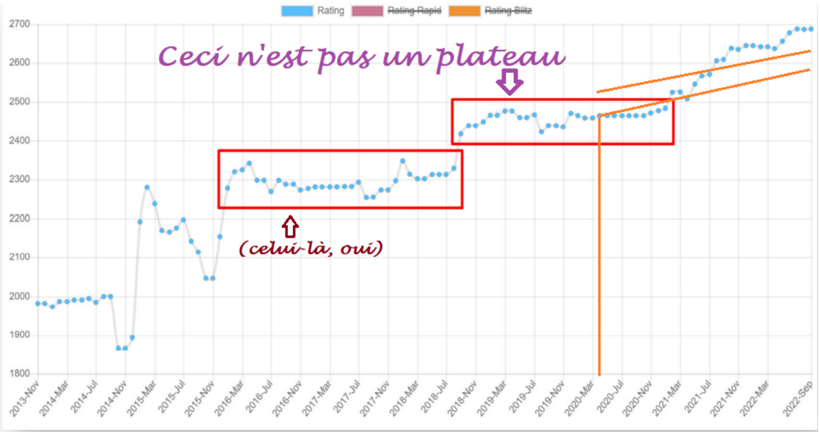
Example Application and Reasoning

Suppose one gets a z-score of **4.00**.

- The **primary meaning** is that the performance has a natural frequency of about **1-in-31,574**, for that quality or higher.
- Let's round that to what I call "Face-Value Odds" of 30,000-to-1.
- This needs to be rectified according to various factors:
  - The **prior likelihood** of cheating. In-person: 1-in-5,000 to 1-in-10,000? Online: 1-in-50 to 1-in-100. :-(
  - The **look-elsewhere effect**: How many others could you have tested? How many in the tournament? How many others playing comparable-level chess that weekend? week? month? year?
- Presence of **other, non-quality evidence** offsets these matters.
- OTB, divide 30,000 by 10,000 leaves just a "balance of probability." Insufficient. Need $z \geq 5$ for comfort.
- Online, dividing by 100 leaves 300-to-1 "reckoned odds" against the *null hypothesis* of fair play.
- Interpret 100-1 to 1,000-1 as range of **comfortable satisfaction** per CAS Lausanne.

Images de "Tricherie"? Graph eines Niemann

The #1 scientific role I've played during the pandemic has been estimating the true skill growth of young players while their official ratings have been frozen.



(The initial talk (10 minutes before Q&A) ended here...)

Two items of larger scientific significance:

1. I have accepted lower sensitivity and predictivity in order to preserve *explainability* and gain *robustness*. Neural methods have been brittle in ways discussed here and here. I present a recent instance linked in an Update at the bottom of this GLL blog post.
2. I suspect that model designers often *satisfice*. That is, they design a model for one purpose but do not sufficiently explore the neighboring problem space for proof against "mission creep" or situational data bias, nor invest in cross-validation. I intend to criticize this study, whose results I do not reproduce.

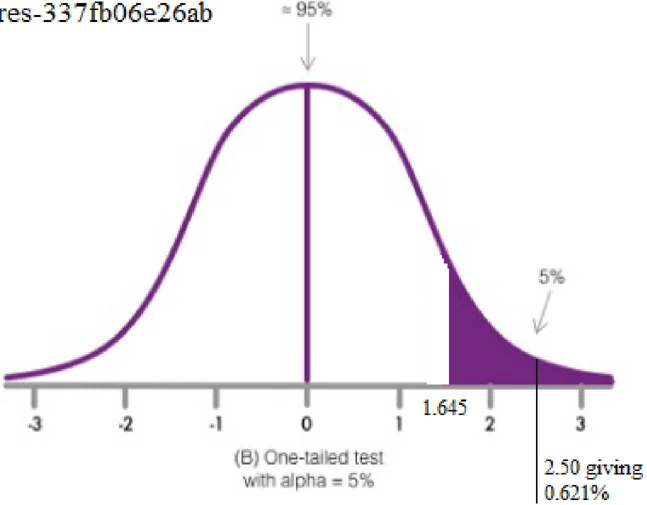
Z-scores

For **independent** situations whose results add up, one can replace probabilities by **Z-scores**, which quantify deviations of averages from expected means.

- Like how raw numbers are indexed by their logarithms on a slide rule.
- A z-value denotes the natural frequency of *at least* yea-much deviation.
- In our homes and flooding example :
  - $z = 2$ indexes the probability that **15** *or more* homes get flooded. About **1-in-44**, which is somewhat under 2.5% probability.
  - $z = 3$ means at least "**17.5**" homes being flooded, 1-in-741 frequency.
  - $z = 4$ means **20** or more flooded, for **1-in-31,575** frequency. (Ignoring that "half a home" matters here too.)
  - $z = 6$ means **25** or more. A "Six-Sigma Deviation": 1-in-a-billion.
- Like with a **Richter Scale**, +1 matters a lot.

Bell Curve and Tails

From https://towardsdatascience.com/hypothesis-testing-z-scores-337fb06e26ab



Central Limit Theorem and "Rule of 30"

Theorem (CLT)

For **any** *probability distribution D, the mean of N* **independent** *samples from D is distributed more like the bell curve as $N \to \infty$.*

- Origin in the accuracy of N trials of any scientific measurement.
- **Convention**: closeness to bell curve "kicks in" at $N = 30$.
- Shadable either way. My latest doctoral student used 3 sets of $N = 15$.
- In chess, the distribution D isn't the same for different chess positions.
- But it stays "chessy." I'm fully comfortable with $N = 50$.
- For screening test, prefer $N = 100$ (usually 4 games).

Using Z-Scores

- Golf-shot analogy for why one uses the whole tail.
- The common "sigma" units allow combining z-scores of disparate events.
- The z-value gives "Face-Value odds" against the *null hypothesis* of the deviation occurring by natural chance.
- **z = 2.00: 1-in-44** odds, 2.275% natural frequency.
- **z = 3.00: 1-in-741** odds, 0.135% natural frequency.
- **z = 4.00: 1-in-31,574** odds, 3.167/100,000 natural frequency.
- **z = 5.00: 1-in-3,486,914** odds, 2.87/10,000,000 natural freq.
- But face-value odds need to be tempered against Bayesian priors, the look-elsewhere effect, and possible selection bias.

Extremes, Dependence, and Adjustments

Going back to our homes-and-flooding example:

- All 100 homes being flooded gives $z = 18$. Beyond astronomical.
- But what if all 100 homes are together and a big storm comes?
- Problem is the home risks not being independent.
- Chess "homes" are like spaced 10km apart in a straight line from Kyushu to Hokkaido.
- "Sparse dependence" with exponential decay within a game.
- Book between games is removed already.
- Can approximate effect of *covariance* by adjusting z 10–15% downward.
- These are my **adjusted z-scores**.
- Both determined and vetted by millions of *resampling* trials—emphasizing 4-game, 9-game, and 16-game sets.

Sensitivity, Soundness, and Safety

- Model is *sensitive* if whenever there is a high deviation in fact, the model registers a high z-score.
- Also termed: the model avoids *false negatives* / avoids *type-2 errors*.
- Model is *sound* if whenever it measures a high z-score there is a factual high deviation.
- Aka.: avoids *false positives* / avoids *type-1 errors*.
- Model is *safe* if in the absence of systematic deviations, the z-scores it gives follow a normal distribution—or at least are *conservatively* within the $z \geq 2$ high end of the standard bell curve.
- It is possible for models to be safe without being sensitive.
- My model has preserved safety while improving sensitivity.
- Safe models can still give false positives in (*normally rare*) cases.

Interpreting Results I.

- Suppose we get $z = 4$. Natural frequency is 1-in-31,574.
- Can we conclude 31,573-to-1 odds that the result is *unnatural* (i.e., cheating)? *Not so fast.*
- Interpretation needs **Bayesian** reasoning about the **prior rate** of cheating.
- If no one could possibly be cheating, it *must* have been a rare but natural event.
- If several cheaters have already been found, chances are you caught another.
- If this is **1** anomaly in a **500**-player Open, *hmm...*
- Context Matters, unfortunately...
- ...or *fortunately*—even in quantum mechanics, the basic working of Nature. Or at least in population medicine...

Cancer and Covid (= in-person and online chess)

- Say you take a test that is **98%** accurate for a cancer that affects 1-in-5,000 people...
- ...and get a positive. *What are the odds that you have the cancer?*
- Not the same as the odds that any one test result is wrong.
- Consider giving the test to 5,000 people, including yourself.
  - Among them, **1** has the cancer; expect that result to be positive.
  - But we can also expect about **100** false positives.
  - All you know at this point is: you are **one** of **101** positives.
- So the odds are still 100-1 against your having the cancer.
- The test result knocked down your prior 5,000-to-1 odds-against by a factor of 50, but not all the way. Need a "Second Opinion."
- IMPHO, 1-in-5,000 $\approx$ frequency of cheating in-person.
- A positive from a "98%" test is like getting $z = 2.05$. *Not enough.*
- In a 500-player Open, **you should see ten such scores**.

The 99.993% Test

- Suppose our cancer test were 600 times more accurate: **1-in-30,000** error.
- That's the face-value error rate claimed by a $z = 4$ result.
- Still 1-in-6 chance of false positive among 5,000 people.
- (This is really how a "second opinion" operates in practice.)
- If the entire world were a 500-player Open, then 1-in-60 chance of the result being natural.
- Still not **comfortable satisfaction** of the result being unnatural.
- IMPHO, the interpretation of CAS comfortable-satisfaction range of **final odds** determination is **99%–99.9%** confidence.
- Target confidence should depend on gravity of consequences. (CAS)
- Sweet spot IMHO is **99.5%**, meaning **1-in-200** ultimate chance of wrong decision. Same criterion used by **Decision Desk HQ** to "call" US elections.
- Higher stringency cuts against timely public service.

Covid in Non-Surge and Surge Times

- Now suppose the factual positivity rate is **1-in-50**.
- We still have about **100** false positives, but now also **100** factual positives.
- A positive from a 98% test is here a 50-50 coinflip.
- But a negative is *good*:
  - Only 2 false negatives will expect to come from the **100** dangerous people.
  - From the **4,900** safe people, about **4,800** true negatives.
  - Odds that your negative is false are **2,400-to-1** against.
- *Fine to be on a plane.* What happened is that the 98%-test result multiplied your confidence in not having Covid by a factor of almost 50.
- **Now suppose the factual positivity rate is 20%.** Can we do this in our heads?

Back to Chess...

- Suppose we get $z = 4$ in online chess with adult cheating rate 2%.
- Out of 30,000 people:
  - **1** false positive result.
  - **600** factual positives.
  - So **600-1** odds against the null hypothesis on the $z = 4$ person.
- A $z = 3.75$ threshold leaves about **200-1** odds. OK here, but not if factual rate is under 1%.
- This analysis does not depend on how many of the factual positives gave positive test results.
- If test is only 10% sensitive, then we will have only about 60 positive results. It sounds like the 1-in-60 case. But the chance of getting a $z = 4$ result on the 1 brilliant player also *generally* goes down to 1-in-10. The confidence ratio is $60/0.10 = 600$-to-1 even so.
- *Sensitivity and soundness generally remain separate criteria.*
- This is relevant insofar as I often get a lot of 3.00–4.00 range results.

Interpretations II: Multiple Factors

- Online platforms collect data on player behavior: clicks, changes in window focus, timing of moves.
- Independence is relative to profiled tendencies.
- For repeated actions, CLT applies, so deviations can be expressed via z-scores.
- If you get z_1 from quality metrics and z_2 from the interface ("telemetry"), weight these factors equally, and consider them independent, then the overall z-score is

$$z = \frac{z_1 + z_2}{\sqrt{2}}.$$

- (If you give weights w_1, w_2 then the formula is $z = \frac{w_1 z_1 + w_2 z_2}{\sqrt{w_1^2 + w_2^2}}$.)
- E.g., if both z_1 and z_2 are 3.5 then $z = \frac{7.0}{1.414...} \simeq 4.95$.
- Face-value odds about 1 in 2.7 million, enough for "any" prior.

Interpretations III: Other Distinguishing Marks

Suppose we have one of thse two situations with player giving $z = 4$:

(a) Player found with cellphone on person.

(b) Player stowed cellphone in bag under chair, switched off [but it still rang].

- In (a), there do not exist 31,574 or even 500 players who do this normally (in any year).
- Can sanction for violation of rule in any event.
- Far more likely that $z = 4$ means cheating. The false-positive guy under this combination won't arise in 60 years.
- Logic goes for $z = 3$ and $z = 2.75$ and even $z = 2.5$ (1-in-161 frequency).

But in situation (b), it matters *how many* players do it, and whether it is *neutral* or *material*.

Distinguishing Marks, continued

- If (b) is also material (or otherwise "covariant") with cheating, then I argue the face-value odds from the z-score become true odds, same as in situation (a).
- Even if (b) is *neutral*, still a problem if:
  - the behavior is infrequent, *and*
  - we are not keeping a large catalogue of arbitrary/impertinent behaviors.
- Suppose only 1,000 players do (b) in any year.
- Then the false-positive guy for $z = 4 \wedge (b)$ comes only once per 31.5 years.
- So **30-to-1** odds against this year—especially if this is the first year of the policy.
- Not enough for comfortable satisfaction, but $z = 4.265$ gives 1-in-100, $z = 4.42$ gives 1-in-200 (round number $z = 4.5$).

Distinguishing Marks, continued

- Suppose it's (b'): player wears green sneakers.
- Less frequent but completely neutral, arbitrary, impertinent.
- Judging based on that would be *selection bias*.
- How about (b''): player wears heavy sweater in hot June weather?
- Together with $z = 3.29$, how the case alluded to in my "Doomsday Argument in Chess" article stood.
- The low frequency—maybe at most 10 players per year do this?—does influence whether material.
- But even if *neutral*, at 1-in-2,000 face-value odds, the false positive for this combination comes once every **200** years.
- If we have a catalogue of **10** things like this, we err once in **20** years.
- (As it happens, my sharper August 2019 model gave some $z > 5$ readings, then more games were found which made $z > 6$ overall.)