Quantum Computing Research Directions

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19 March, 2019

[1] Joint work with Amlan Chakrabarti, U. Calcutta, and Chaowen Guan, UB.

Algebraic and Logical QC Simulations

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- Partly included in textbook with Richard Lipton, *Quantum Algorithms Via Linear Algebra* (MIT Press, 2014; 2nd. ed. to come this year).

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- Also give $O(N)$-time reduction $(N = n^2)$ from computing $n \times n$ matrix rank over $\mathbb{F}_2$ to the QC simulation.
- Means that the n^2-vs.-n^ω weak/strong simulation gap canot be closed unless matrix rank is in $O(n^2)$ time over $\mathbb{F}_2$.

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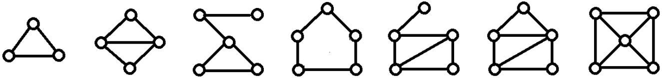
- Also yields an apparently new class of undirected graphs:



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- Second main purpose of simulator [show] is to enable tinkering with approximative methods.

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- Also apply to study T-*gate count*, singular points. . .

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- Avenues for dissemination: *Gödel's Lost Letter* blog, textbook with MIT Press going to second edition this summer.
- Involvement in the general debate over *Quantum Advantage*.