Time and Difficulty RIT Colloquium

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A Predictive Analytic Model

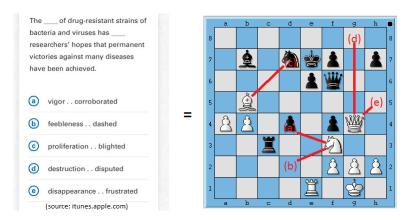
Means that the model:

- Addresses a series of events or decisions, each with possible outcomes $m_1, m_2, \ldots, m_j, \ldots$
- Assigns to each m_j a probability p_j .
- Projects risk/reward quantities associated to the outcomes.
- Also assigns confidence intervals for p_j and those quantities.

In a *utility-based* model, each m_i has a utility or cost u_i . The main risk/reward quantity is then $E = \sum_i p_i u_i$. Examples:

- Insurance: m_i are risk factors; costs u_i do not influence p_i .
- Chess: m_i are legal moves; u_i are values given by strong chess-playing programs that objectively say how good the moves are. In my model, p_i depend on u_i per bounded rationality.
- Multiple-choice tests: m_i are possible answers to a test question, $u_i = \text{gain/loss}$ for right/wrong answer.

Chess and Tests—With Partial Credits (Or LLMs?)



Here (b,c) are equal-optimal choices, (a) is bad, but (d) and (e) are reasonable—worth part credit.

Move Utilities Example (Kramnik-Anand, 2008)

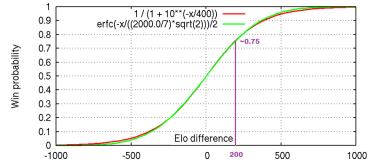




	Depti	1S													Values by Stockfish 6					
Move	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
Nd2	103	093	087	093	027	028	000	000	056	-007	039	028	037	020	014	017	000	006	000	
Bxd7	048	034	-033	-033	-013	-042	-039	-050	-025	-010	001	000	-009	-027	-018	000	000	000	000	
Qg8	114	114	-037	-037	-014	-014	-022	-068	-008	-056	-042	-004	-032	000	-014	-025	-045	-045	-050	
Nxd4	-056	-056	-113	-071	-071	-145	-020	-006	077	052	066	040	050	051	-181	-181	-181	-213	-213	

Aptitude—Via Elo Grades (calculator)

- Named for **Arpad Elo**, number R_P rates skill of player P.
- E.g. 1000 = bright beginner, 1600 = good club player, 2200 = master, 2800 = world championship caliber.
- Computer engines are far higher, e.g.: Stockfish 16 = 3544, Torch 1.0 = 3531, Komodo Dragon 3.3 = 3529.
- Expectation $e = \frac{1}{1 + \exp(c(R_P R_O))}$ depends only on difference to opponent's rating R_O . With $c = (\ln 10)/400$ the curve is:



Main Parameters and Inputs

The (only!) player parameters trained against chess Elo Ratings are:

- s for "sensitivity"—strategic judgment. Like Anatoly Karpov.
- c for "consistency" in tactical minefields. Like Mikhail Tal.
- h for "heave" or "Nudge"—obverse to depth of thinking.

Trained on all available in-person classical games in 2010–2019 between players within 10 Elo of a marker 1025, 1050, ..., 2775, 2800, 2825. Wider selection below 1500 and above 2500.

- Given an Elo rating R, "central slice" gives corresponding s_R, c_R, h_R .
- Only other input is move values at various depths of search.
- Important "differentiator": my heavily scaled version (ASD) of "average centipawn loss."
- Other than these, my model knows nothing about chess.

Log-Linear Versus Loglog-Linear Model

The generic log-linear model puts

$$\log\left(\frac{1}{p_i}\right) = \alpha + \beta u_i$$
, or equivalently, $\log\left(\frac{1}{p_i}\right) - \log\left(\frac{1}{p_1}\right) = \beta \delta_i$,

where $\delta_i = u_1 - u_i$. Solved by **softmax** giving $p_i = p_1 \exp(-\beta u_i)$, so each p_i is represented as a **multiple** of the best-move probability p_1 .

The **loglog-linear** model puts $\log \log(\frac{1}{p_i}) - \log \log(\frac{1}{p_1}) = \beta \delta_i$, i.e.:

$$\frac{\log(1/p_i)}{\log(1/p_1)} = \exp(\beta \delta_i).$$

This gives $p_i = p_1^{\exp(\beta \delta_i)}$, so probabilities are represented as **powers** of p_1 .

In place of $\beta \delta_i$, I have $\left(\frac{\delta_i - h\rho_i}{s}\right)^c$, where the "heave term" ρ_i uses the values at lower depths of search. Why h is tightly clamped.

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How it Works

- Take s, c, h from a player's rating (or wider skill profile).
- Generate probability p_i for each legal move m_i .
- Paint m_i on a 1,000-sided die, 1,000 p_i times.
- Roll the die to give confidence intervals that go with the p_i .
- (Correct after-the-fact for chess decisions not being independent.)

Main Outputs:

- Statistical z-scores for various (actual-projected) quantities:
 - T1-match: Agreement with the move listed first by the computer.
 - EV-match: Includes moves of equal-optimal value not listed first.
 - **ASD**: Average *scaled* difference in value from inferior moves.
- An Intrinsic Performance Rating (IPR) for the set of games.

Karpov & Tal at Montreal "Tourney of Stars" 1979

- Tied for first with 12/18 in star-studded double round-robin.
- Karpov was rated 2705, Tal only 2615.
- Karpov (per Stockfish 11): s = 0.016, c = 0.307.
- Tal (per Stockfish 11): s = 0.026, c = 0.365.
- Lower s is better—so Karpov was more "Karpovian."
- ullet Higher c is better—so my model with Tal's parameters would make fewer large mistakes.

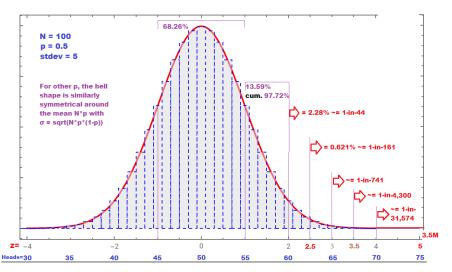
Are these grainy parameters enough to mimic human tendencies?

- IPRs: Karpov 2625 +- 155, Tal 2730 +- 185.
- Whole tourney IPR is (only!) **2575** +- **50** (s = 0.041, c = 0.385).
- Average Elo of players, **2621**, is within error bars. Surprise is that the IPR is not near 2700s range. Today's elite regularly hit 2800+.

Z-Scores

- A **z-score** measuresf performance relative to natural expectation.
- Used extensively by business in Quality Assurance, Human Resources Management, and by many testing agencies.
- Expressed in units of standard deviations, called "sigmas" (σ) .
- Correspond to statements of odds-against (but see next slides):
- "Six Sigma" (6σ) means about 1,000,000,000–1 odds;
- $5\sigma = \text{about } 3,500,000-1;$
- $4.75\sigma = \text{about } 1,000,000-1;$
- $4.5\sigma = \text{about } 300,000-1;$
- $4\sigma = \text{about } 32,000-1;$
- $3\sigma = \text{about } 750-1 \text{ (closest is } 740-1);$
- $2\sigma \doteq 43$ –1 (civil minimum standard, polling "margin of error").

Bell Curve and Tails (also Screening Stage)



Suppose We Get z = 3.54

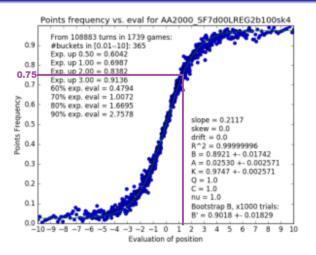
- Natural frequency \approx 1-in-5,000. Is this Evidence?
- Transposing it gives "raw face-value odds" of "5,000-to-1 against the null hypothesis of fair play. **But:**
- Prior likelihood of cheating is estimated at
 - 1-in-5,000 to 1-in-10,000 for in-person chess.
 - 1-in-50 (greater for kids) to 1-in-200 for online chess.
- Look-Elsewhere Effect: How many were playing chess that day? weekend? week? month? year?

Are these considerations orthogonal, or do they align?

If you're "marked" by a previous incident, these recede.

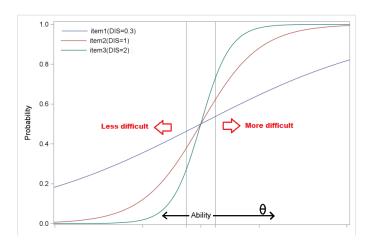
If there is on-site evidence, z = 2.50 is enough (FIDE).

Position Value \longleftrightarrow Expectation (2000 vs. 2000)



- Similar 0.75 expectation when up 1.30 vs. equal-rated player.
- Complication: dependence on rating itself.

Item-Response Theory (IRT source)



- Horizontal axis governs difficulty in relation to $\theta = ability$.
- Slope at y = 0.5 correctness rate is the discrimination factor.

Defining Difficulty

- For any fixed aptitude level θ , difficulty \approx expected points loss.
- In chess, this is our $E_L = \sum_i p_i(u_1 u_i) = \sum_i p_i \delta_i$.
- Call this expected loss the hazard.
- Depends on rating because the probabilities p_i projected by my model depend on rating R.
- My model divides out dependence on R. "Expectation Weights, Normalized" (EWN).
- Technotes: In a log-linear model, $-\log p_i \sim u_i$.
- Then $E_L \sim \sum_i p_i \log(1/p_1) \sum_i p_i \log(1/p_i) = \log(\frac{1}{p_1}) H$ where H is **entropy**.
- *However*, my model is **double-log linear**: $\frac{\log p_i}{\log p_1} \sim \exp(\delta_i)$.
- Why double-log works and single-log fails.
- How well does hazard—normalized over aptitude—work as a measure of difficulty?

A Philosophical Issue

Should a grading metric μ expect to assess lower performance on more-difficult questions, or should it show a *constancy of signal* θ across all types of questions?

- I typically design exams to have 20% A-level questions, 30% B-level, 30% C-level, 20% D-level.
- Overall threshold for A: 90%.
- Getting 60% on the A-level questions puts you on-track, even though 60% by itself is C-range (or worse).
- Thus the simple grading score μ does not give constant signal—it needs context.
- Should we use metrics that say "A-level" etc. in each category? (Like *curving*).

Model and Metrics

The following "raw metrics" on series of games are used generally:

- T1-match: Agreement with the move listed first by the computer.
- EV-match: Includes moves of equal-optimal value not listed first.
- **ASD**: Average difference in value from inferior moves (over all positions), but *scaled* down when one side has advantage.
 - Called **ACPL** for average centipawn loss without scaling.

All should vary with difficulty, hence not give constancy of signal.

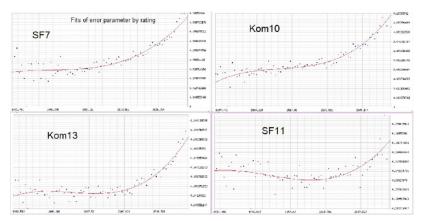
- My Intrinsic Performance Rating (IPR) metric fits parameters
 - s for "sensitivity" (\sim strategic ability), and
 - c for "consistency" (in surviving tactical minefields)

to give the closest $Virtual\ Player\ P(s,c)$ on any set of games.

- Then trained correspondence $(s,c) \to R$ gives IPR as an Elo rating.
- Should give constancy of signal...but...

How Accurate Are Model Projections?

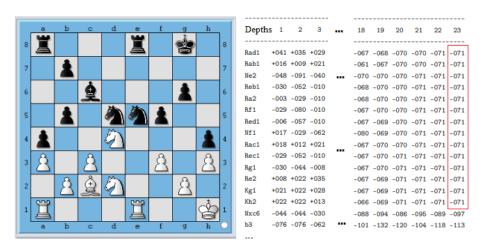
Internal evidence that it gives $\approx (1 + \epsilon)$ relative error with $\epsilon \approx 0.04$ for most rating levels. Means it supports betting on chess moves with only 5% "vig" to avoid arbitrage. (Except for bets against clear-best moves.)



IPR and Hazard (World Senior Teams 2024)

- Older players, established ratings (but deflated), average **2080**.
- Focus on 2000–2200. Analysis by Stockfish 11 in EWN mode.
- IPR overall: **2125** +- **40**. Broken down according to [dis-]advantage:
 - 1–2 pawns behind: **2170** +- **105**; worse: **2065** +- **110**.
 - 1–2 pawns ahead: **2085** +- **120**; better: **2020** +- **155**
 - Within 1.00 of equal: **2145** +- **45**; within 0.50: **2125** +- **65**.
- Reasonable constancy of signal.
- But on positions with ≥ 1.5 times normal hazard: **2255** +- **65**.
- With $\geq 2x$ hazard: 2170 +- 115. Could be consistent. But—
- Positions of of 0.5x or lower hazard: 1800 +- 180.
- Not constancy of signal.
- Low-hazard positions either have an obvious best move or many good moves.

Example: Niemann-Shankland, USA Ch. 2023



Low-hazard because crisis is far off, but difficult in real chess terms. Low E_L , high entropy H. (Niemann lost.)

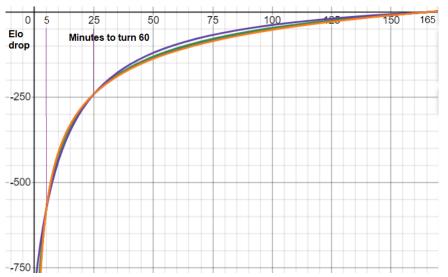
Aspects of Difficulty (Besides Hazard)

- Needing deep cogitation to find best move or avoid a trap. Expressly modeled—e.g. to project the trap for Kramnik.
- Being at a disadvantage. Chess, not so much examinations.
 Model performs fine.
- **4** Humans perform poorly. Basic with repeatable test questions. Repeatable chess positions, however, are opening book knowledge.
- Humans take a long time to answer.
 - Can't project ahead of time (owing to non-book \equiv non-repeatable).
 - But certainly directly captures the human *experience* of difficulty.
- **1** Question is inherently complex or taxing.
 - How to measure this internally?
 - Sunde, Zegners, and Strittmatter [SZS, Jan. 2022] propose counting the time (i.e., number of position nodes) needed by chwess engine to complete analysis to depth (say) 24.
 - Carow and Witzig [CW, Feb. 2024] consider all the above, but strive for human-chess based measures.

Time Budget and Effect on Quality

- FIDE Standard Time Control: 90 minutes to turn 40, then 30 minutes more, with 30-second *increment* after every move. Allows 150 minutes to turn 60.
- "Standard" control must allow at least 120 minutes to turn 60.
- Some elite events allow 180, 195, even 210 minutes (to turn 60).
- Rapid means any time giving under 60 minutes and at least 10. Common is 15 min. plus 10-second increment, giving 25 to turn 60.
- Blitz means under 10 minutes, most common is 3 minutes + 2-second increment, which gives 5 minutes—and so approximates old-school 5-minute chess on analog clocks.
- For 25-minute Rapid, I measure **240** reduction in quality per IPR.
- For 5-minute Blitz, 575 lower. (Error bars for both are about ± 25 .)

Time-Quality Curves (whole graph)



Predicated on Time Spent For a Move

Staying with players rated 2000 to 2200 at the World Senior Team Ch.

- Positions on which they spent at most **30 seconds** on the move: **2860** +- **75**.
- At most 10 seconds: 3235 +- 90.
- Starting at turn 16 rather than 9: **3220** +- **100**.
- At most 5 seconds (sample size 605): 3230 +- 160.

What gives here? How about moves with long thinks—?

- Positions with 5–10 minutes consumed: **1460** +- **85**.
- Using 10–15 minutes (705 positions): **1235** +- **170**.
- Using ≥ 15 minutes (371 positions): **1410** +- **205**.
- "Thinking Is Bad For You." (At least it's a bad sign...)
- Vivid reproduction of [SZS 2022] (and also Anderson et al., 2016 thru now for online blitz).



Instead of Seniors, Let's try 8-Year-Olds!

After 3 rounds of the ongoing **2024 World Cadets Championships** in separate Open and Girls' sections of ages **U08**, **U10**, and **U12**.

- The two **U08** sections combined have average rating 1596.
- I measure IPR as **1525** +- **45**. (10,913 positions total)
- In EWN mode, **1490** +- **65**.
- Positions on which they spent at most **30 seconds** on the move: **2170** +- **125** (2,996 pos.)
- At most **10** seconds: **2860** +- **245** (632 positions)
- At most **5** seconds (sample size 151): **2935** +- **555**.

How about when little kids think longer?

- Positions with 5–10 minutes consumed (729 pos.): 650 + 235.
- Using 10–15 minutes (168 positions): **465** +- **565**.
- Using ≥ 15 minutes (104 positions): **700** +- **505**.
- "Thinking Is Bad For Kids Too."

Hazard Vs. Time—and Time Left

Switching to Komodo 13.3 in place of Stockfish 11 as analyzing engine:

- Overall IPR of Elo 2000-to-2200 players: **2175** +- **35**.
- Average thinking time over all moves (turns 9–60): 181 seconds.
- IPR on turns of $\leq 0.5x$ hazard: **1635** +- **125**.
- Average thinking time in those positions: 145 seconds.
- IPR on turns of $\geq 2x$ hazard: **2345** +- **125**.
- Average thinking time in those positions: 151 seconds.

Results are more as-expected on turns with little time budget left:

- When player has ≤ 180 seconds left (633 turns): 1540 +- 280.
- Or average ≤ 60 seconds left to turn 40, not counting increment time: 1685 +- 200.
- Or average 30 seconds left to turn 40, counting half the increment time: 1395 +- 425. (In all cases, average hazard.)

Enter Entropy

Students in my CSE702 graduate seminar proposed a measure H_U of entropy that uses only the move utilities u_i , not the projected probabilities p_i (nor their logs). Avoids the rating feedback loop.

- Average $H_U = 2.57$.
- Turns with $H_U \leq 2$: avg. time used 88 sec., IPR 2405 +- 100.
- Turns with $H_U \leq 1.5$: avg. time used **72 sec.**, IPR **2485** +- **130**.
- Turns with $H_U \leq 1$: avg. time used **56 sec.**, IPR **2645** +- **165** (lower hazard too).
- Turns with $H_U \leq 0.5$: avg. time used **40 sec.**, IPR **2580** +- **255** (much lower hazard).
- Turns with $H_U \geq 3$: time used 252 sec., IPR 2000 +- 35.
- Turns with $H_U \ge 3.5$ (702 pos.): time 312 sec., IPR 1965 +- 110.
- (No position has $H_U \ge 3.8$. All cases have close to mean hazard.)
- High entropy correlates well with (human experience of) difficulty.
- Much more work to do...



Discussion and Q & A

[And Thanks]

[Possible extra slides for Q & A follow...optional, of course...]

Cognitive Concepts and Conceits

Many results in cognitive decision making come from studies that

- are well-targeted to the concept and hypothesis, but
- 2 have under 100 test subjects...
- 3 ...under simulated conditions...
- ...with unclear metrics and alignment of personal vs. test goals..., and where
- ...reproducibility is doubtful and arduous.

The *chess angle* is to trade 1 against wealth of 2,3,4,5: lots of players and games, real competition, clear goals and metrics (Elo ratings), and not only reproducible but conducive to abundant falsifiable predictions.

Some Accompanying Stances

- Extreme Corner of Data Science—since I need ultra-high confidence on any claim.
- Concern: Data modelers in less-extreme settings satisfice.
- That is, their models are designed up to one particular goal but don't explore much of the harder adjacent metaspace.
- Nonreproducibility, Mission Creep, and Shifting Sands. E.g., I do not reproduce the longer conclusions of this study.
- Cross-Validation...one point of which is:
- How can we distinguish uncovering genuine cognitive phenomena from artifacts of the model?

Some Cognitive Nuggets

- Dimensions of Strategy and Tactics (and Depth of Thinking).
 - But wait—the model has no information specific to chess...
 - Brain seems to register changes in move values as depth increases.
- Machine-Like Versus Human Play
 - Garry Kasparov, as a 2012 Alan Turing Centennial test, distinguished 5 games played by human 2200-level masters from 5 games by engines "stopped down" to 2200 level.
- Relationship to Multiple-Choice Tests (with partial credits)
 - "Solitaire Chess" feature often gives part credits.
 - Large field of **Item Response Theory** (IRT).

Player Development

- Rating Inflation? Deflation?
 - Note low Montreal 1979 IPRs.
 - Even further deflation at the 1986 Men's and Women's Olympiads in Dubai.
 - "Today's players deserve their ratings."
 - Is human performance at chess improving as with physical sports? ...because of computers?
- Growth Curves of Improving (Young) Players.
- How To Manage Time Budget (basically, follow V. Anand!).

Cancer and Covid (= in-person and online chess)

- Say you take a test that is 98% accurate for a cancer that affects 1-in-5,000 people...
- ...and get a positive. What are the odds that you have the cancer?
- Not the same as the odds that any one test result is wrong.
- Consider giving the test to 5,000 people, including yourself.
 - Among them, 1 has the cancer; expect that result to be positive.
 - But we can also expect about 100 false positives.
 - All you know at this point is: you are **one** of **101** positives.
- So the odds are still 100-1 against your having the cancer.
- The test result knocked down your prior 5,000-to-1 odds-against by a factor of 50, but not all the way. Need a "Second Opinion."
- IMPHO, 1-in-5,000 \approx frequency of cheating in-person.
- A positive from a "98%" test is like getting z = 2.05. Not enough.
- In a 500-player Open, you should see ten such scores.



The 99.993% Test

- Suppose our cancer test were 600 times more accurate: 1-in-30,000 error.
- That's the face-value error rate claimed by a z = 4 result.
- Still 1-in-6 chance of false positive among 5,000 people.
- (This is really how a "second opinion" operates in practice.)
- If the entire world were a 500-player Open, then 1-in-60 chance of the result being natural.
- Still not comfortable satisfaction of the result being unnatural.
- IMPHO, the interpretation of CAS comfortable-satisfaction range of **final odds** determination is **99**%–**99.9**% confidence.
- Target confidence should depend on gravity of consequences. (CAS)
- Sweet spot IMHO is 99.5%, meaning 1-in-200 ultimate chance of wrong decision. Same criterion used by Decision Desk HQ to "call" US elections.
- Higher stringency cuts against timely public service.

Covid in Non-Surge and Surge Times

- Now suppose the factual positivity rate is **1-in-50**.
- We still have about 100 false positives, but now also 100 factual positives.
- A positive from a 98% test is here a 50-50 coinflip.
- But a negative is *good*:
 - Only 2 false negatives will expect to come from the **100** dangerous people.
 - From the 4,900 safe people, about 4,800 true negatives.
 - Odds that your negative is false are 2,400-to-1 against.
- Fine to be on a plane. What happened is that the 98%-test result multiplied your confidence in not having Covid by a factor of almost 50.
- Now suppose the factual positivity rate is 20%. Can we do this in our heads?

Back to Chess...

- Suppose we get z = 4 in online chess with adult cheating rate 2%.
- Out of 30,000 people:
 - 1 false positive result.
 - 600 factual positives.
 - So 600-1 odds against the null hypothesis on the z=4 person.
- A z = 3.75 threshold leaves about **200-1** odds. OK here, but not if factual rate is under 1%.
- This analysis does not depend on how many of the factual positives gave positive test results.
- If test is only 10% sensitive, then we will have only about 60 positive results. It sounds like the 1-in-60 case. But the chance of getting a z=4 result on the 1 brilliant player also generally goes down to 1-in-10. The confidence ratio is 60/0.10 = 600-to-1 even so.
- Sensitivity and soundness generally remain separate criteria.
- This is relevant insofar as I often get a lot of 3.00–4.00 range results.

Pre-Check: The "Screening" Stage

- Makes a simple "box score" of agreements to the chess engine being tested and the **scaled** average centipawn loss from disagreements.
- Creates a Raw Outlier Index (ROI) from the raw metrics.
- ROI is on same 0-100 scale as flipping a fair coin 100 times: 50 is the expectation *given one's rating* and 5 is the standard deviation, so the "two-sigma normal range" is 40-to-60.
- Like medical stats except **indexed** to common **normal** scale.
- 65 = amber alert, 70 = code orange, 75 = red. Example.
- Completely data driven—no theoretical equation.
- Rapid and Blitz trained on **in-person** events in 2019. Slow chess trained on in-person FIDE Olympiads from 2010 to 2018.
- Does not account for the *difficulty* of games. That is the job of the full model.

Rating Lag—Natural Versus Systematic

- The #1 scientific role I've played during the pandemic has been estimating the true skill growth of young players while their official ratings have been frozen.
- But this has perforce been **post-normal science**.
- My "back of the envelope" formula held up over two years with only one small revision for preteens.
- Larger revision in Oct. 2022 to curtail projections past Elo 2000 level.
- Would have been more "normal" if comprehensive studies of the career arcs (measured by Elo rating) of young players were to hand.
- Lack of such studies exposed by the controversy over Hans Niemann's rise from 2465 Elo to 2700.
- Show this GLL article including example of Ms. Velpula Sarayu.

Independent Corroboration of Others' Work

- The article's larger subject is a **drastic** proposal by US statistician Jeff Sonas—long used by FIDE—to overhaul chess ratings below Elo 2000—that is, for beginning and amateur players.
- (This is on top of things I've been telling FIDE about ratings above 2000.)
- My own work has been "tinged" by this issue.
- A natural metric **apart** from both my model and Sonas's domain cross-validates his observations and arguments.
- I will now discuss some other applications that these solid foundations enable.

Hans Niemann: Platform or Plateau?



The Gender Gap in Chess

- Is clear: with Judit Polgar retired, there are no women in the top 100 by rating.
- Where/when does it begin?
- How should one begin to address this question?
- What data could corroborate a result—or a proposed explanation?
- Picture emerging from recent youth events...?