Data, AI, and Society Lecture 2

How LLMs Work

Kenneth W. Regan
University at Buffalo (SUNY)

CSE199 Freshman Seminar, Fall 2026

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- We will cover their structure and ingredients—and what implications stem from them.

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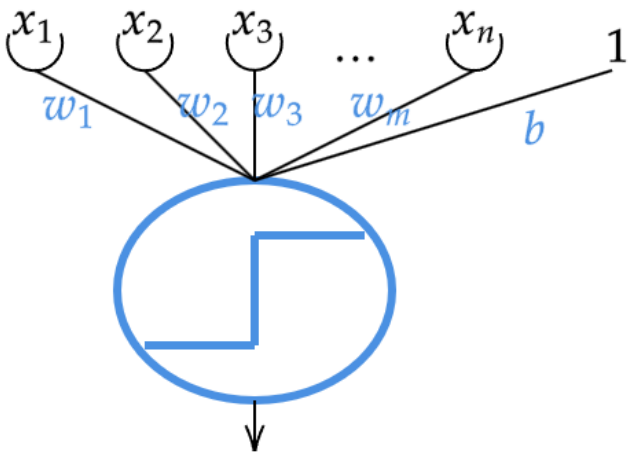
Defining some terms here: A **threshold gate** first computes an affine linear function of some number m of inputs $x_1, \ldots . x_m$:

$$u = w_1 x_1 + w_2 x_2 + \cdots w_m x_m + b,$$

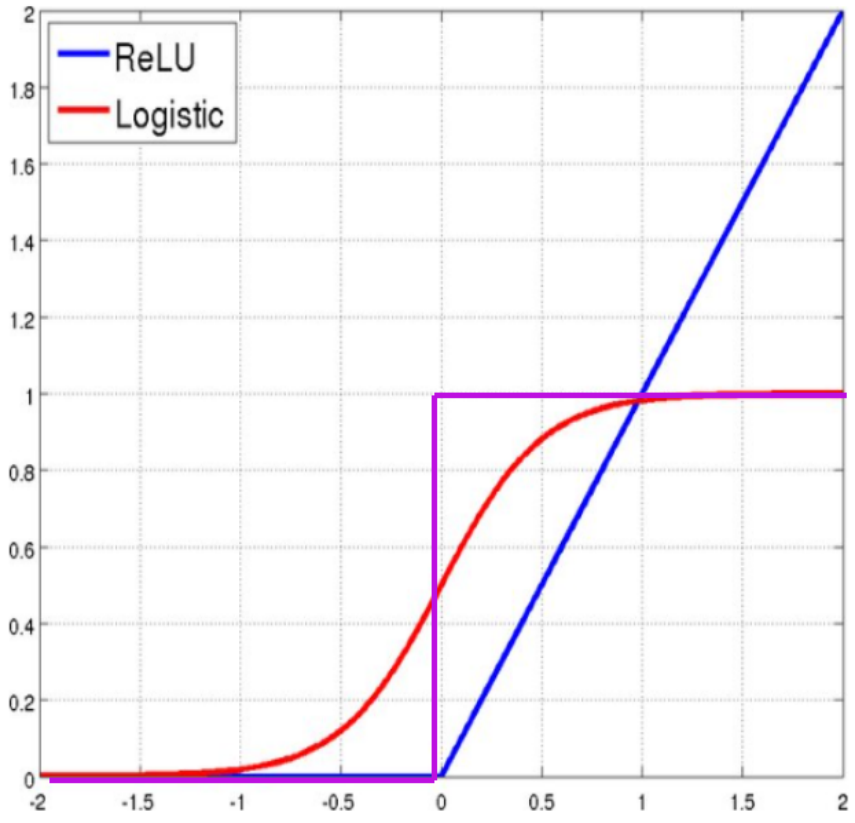
where the **weights** w_i and **bias** b belong to the gate. Then it tests $u > 0$? and outputs an **activation value** $a(u)$ accordingly.

A **perceptron** has just one **layer** of threshold gates that feed into a Boolean AND gate.

Threshold Gate and Alternative Activation Functions



$$\begin{cases} 1 \text{ if } \sum_{i=1}^{m} w_i x_i + b > 0 \\ 0 \text{ otherwise} \end{cases}$$

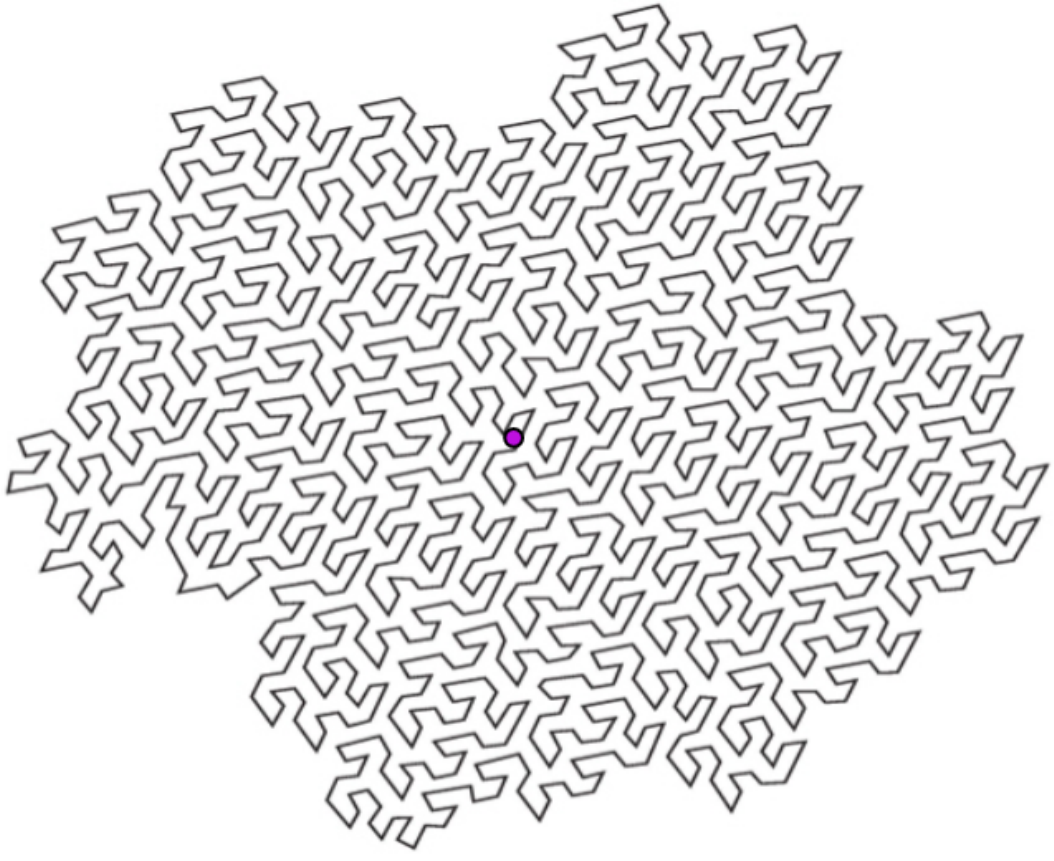


Modified from 2016 Robot World Cup paper Albani et al.,
https://link.springer.com/chapter/10.1007/978-3-319-68792-6_33

Can we keep advantage over AI?—cont'd

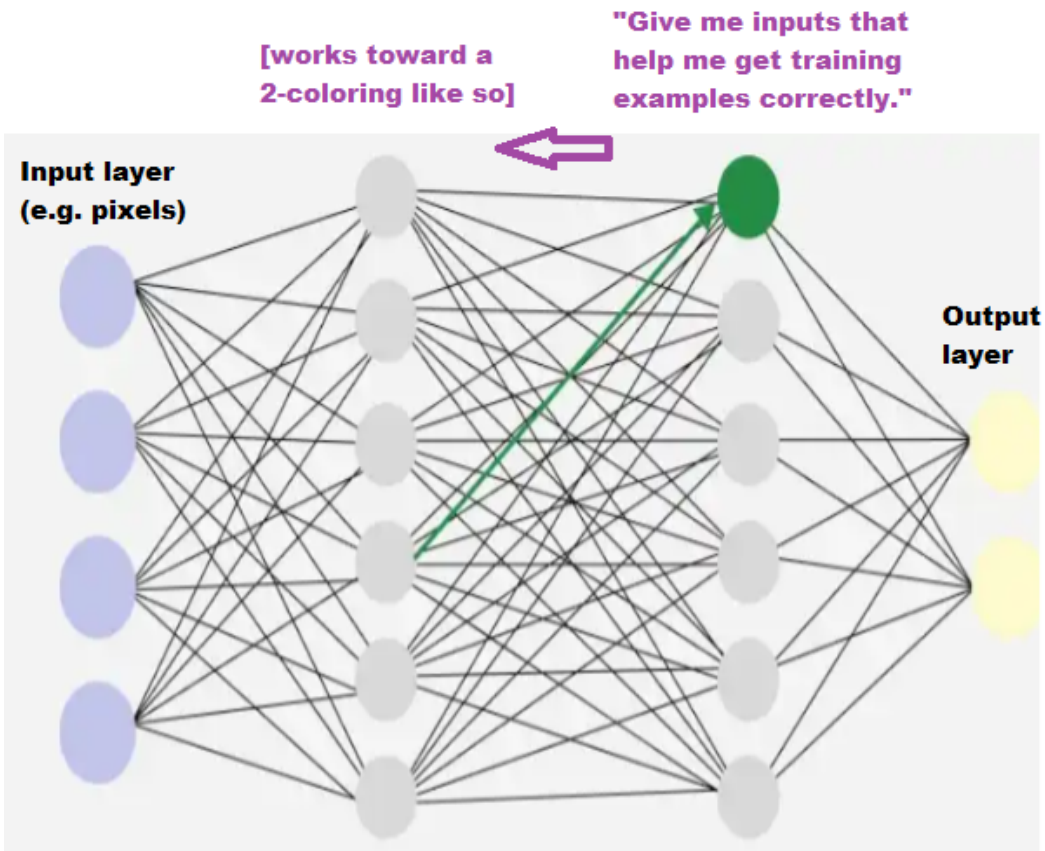
No because our brains cannot readily tell whether the point at right is inside or outside the closed curve---a **parity** task. Nor can a perceptron. This lack was a major cause of the "AI Winter" in the 1970s and 1980s.

Moreover, AI can employ more than one layer of threshold gates. By the 1990s we had algorithms and computing power to train **deep neural networks**.

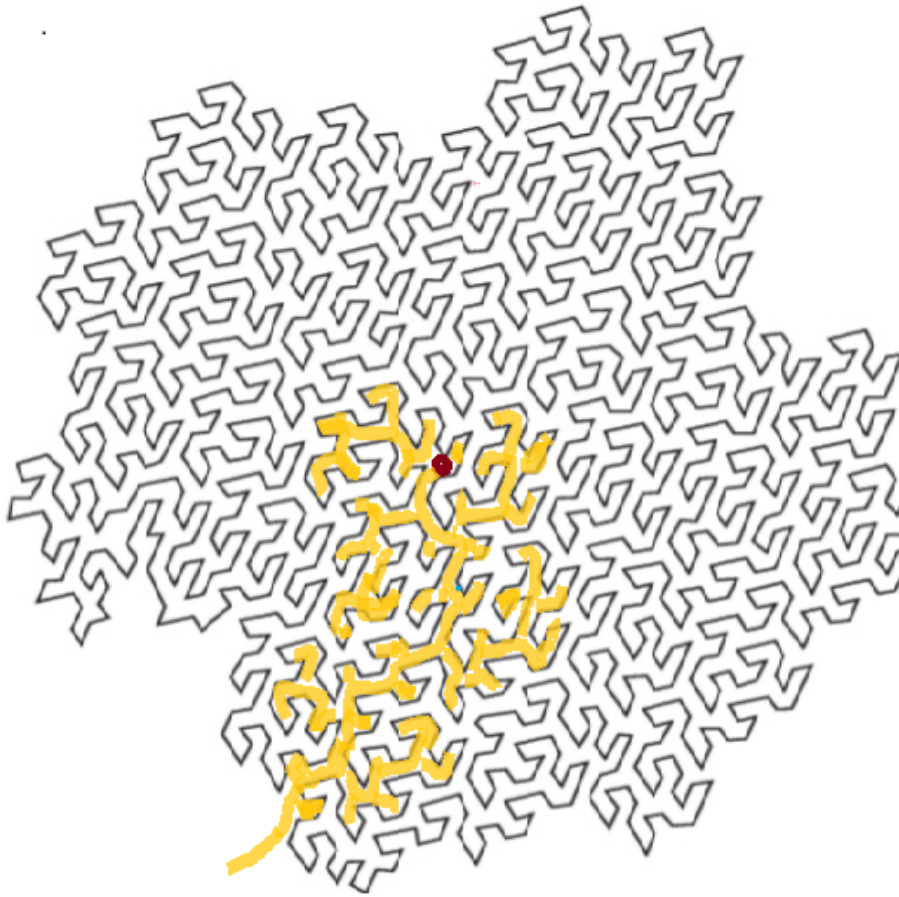


mathoverflow.net/questions/8521/nice-proof-of-the-jordan-curve-theore

Training Multiple Layers: Backpropagation



Modified from https://www.geeksforgeeks.org/deep-learning/layers-in-artificial-neural-networks-ann/
Getting 100% correctness may need a third threshold layer plus a Boolean AND layer.



Gradient descent algorithm minimizes error over any number of layers.

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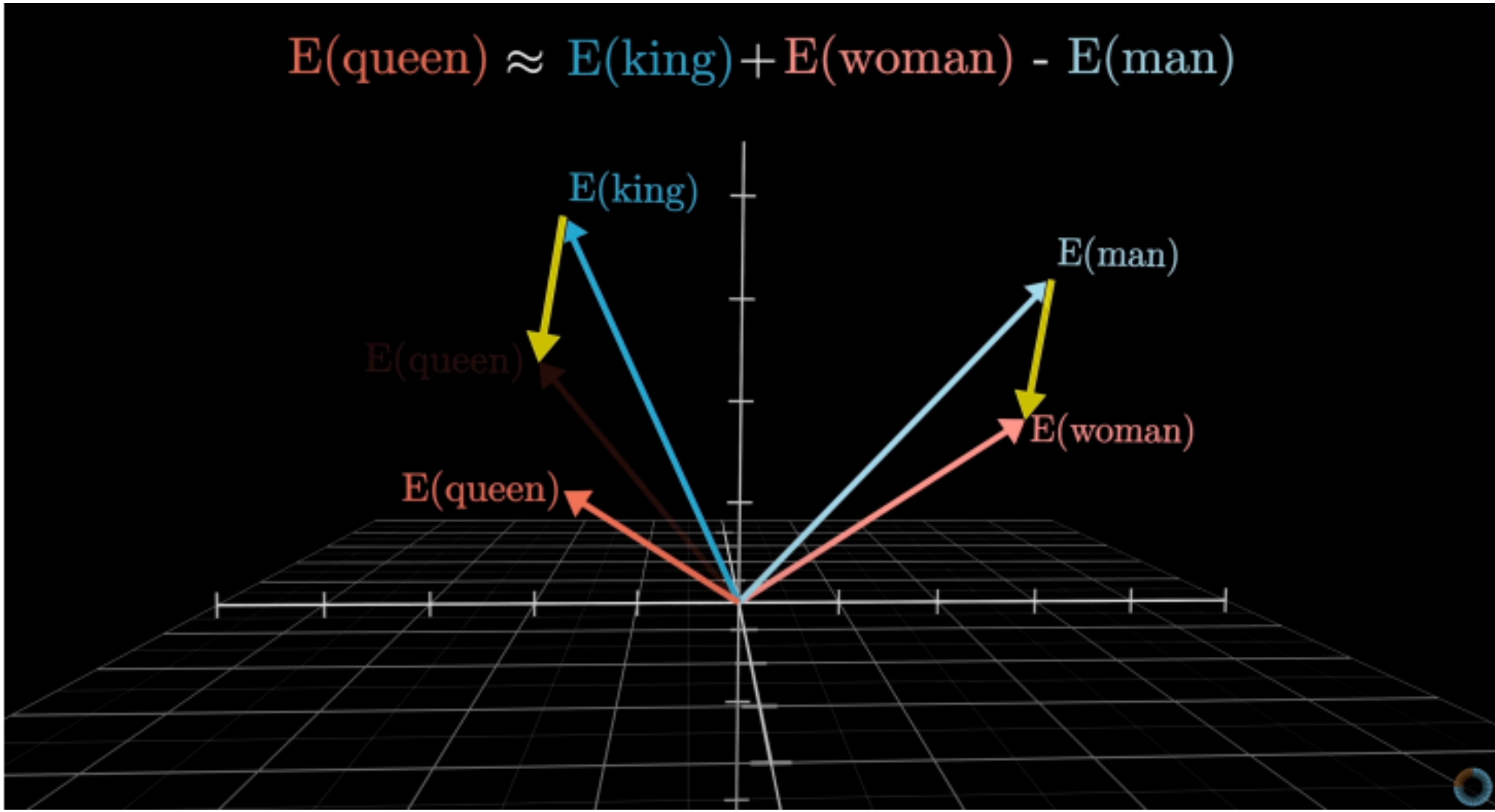
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In color, ♣ and ♠ are black; ♢ and ♡ are red.
In bridge, ♣ and ♢ are "minor"; ♡ and ♠ are "major."

Embedding Example (from 3 Blue 1 Brown)



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- **Efficient to double-up many tokens, with-and-without leading spaces.**
- **Lots of wiggle room**—not only for resolving multiple word meanings by context but also for nuanced shading of meanings.

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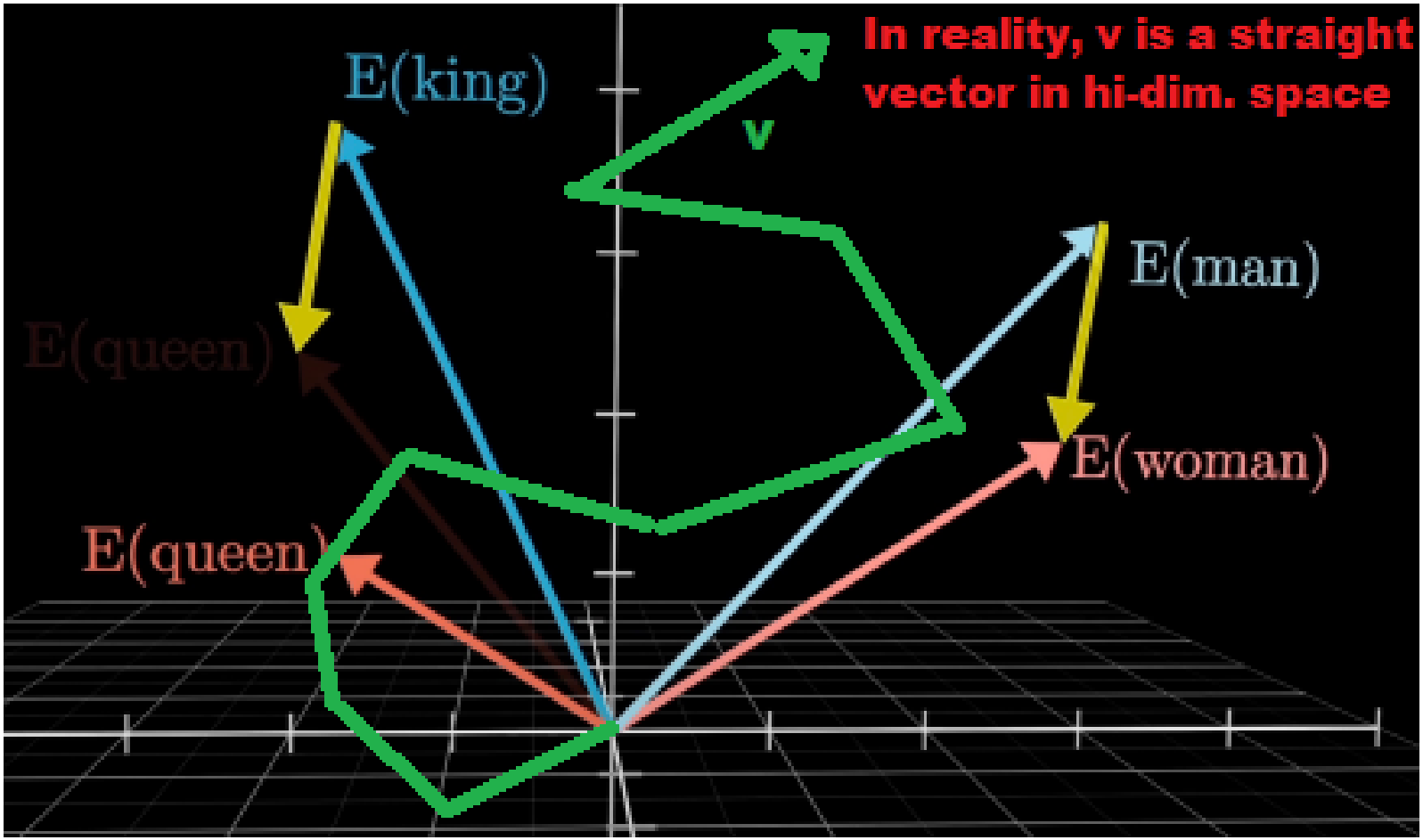
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- Then the new v is updated layer-by-layer.

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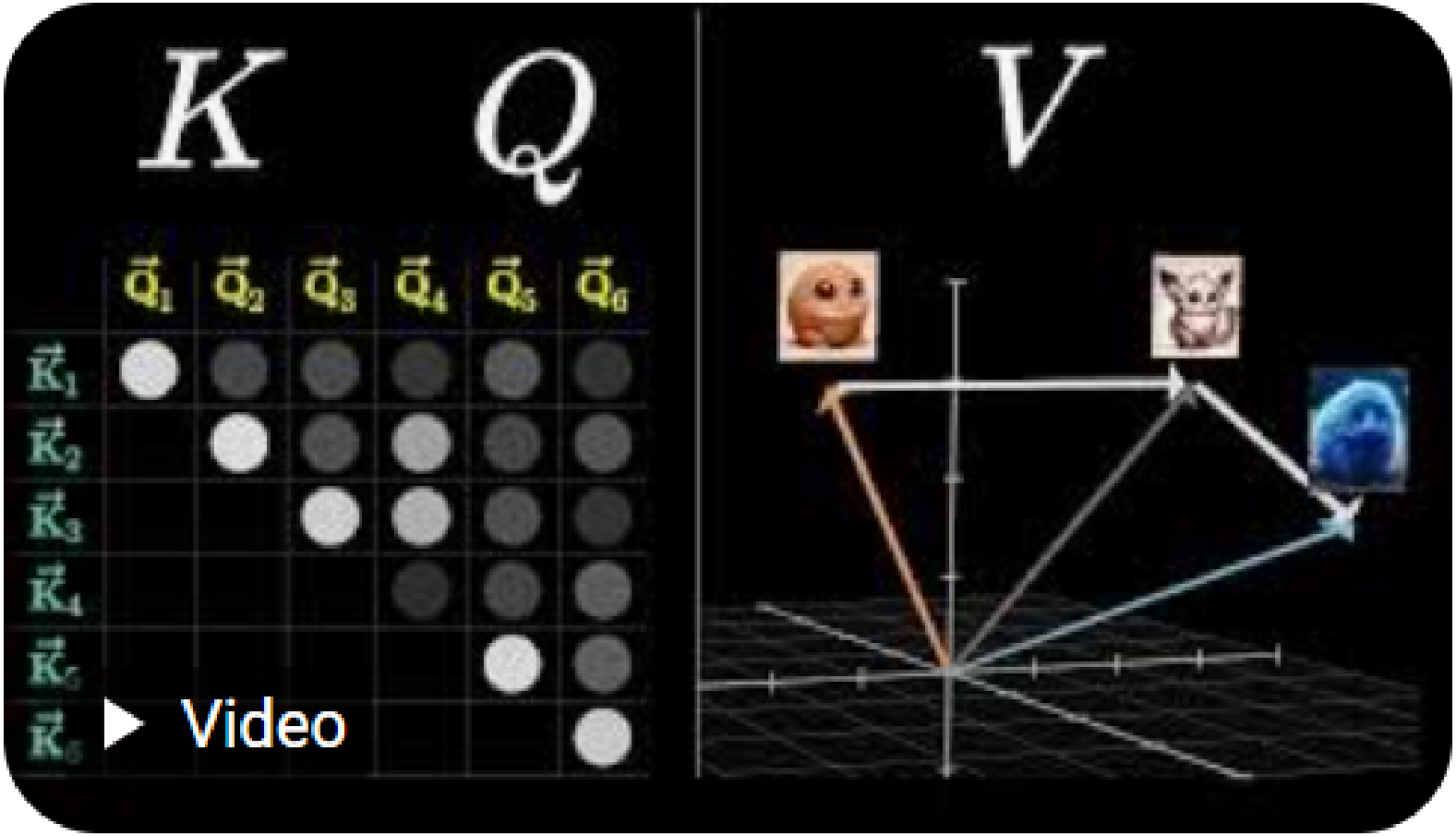
- Candidate next-words can **query** *earlier* words in a given text (which are known by that point)—up to a **context window** size T.
- T is generous now: 32K tokens midrange, 1M for Qwen-3.7 Max.
- Earlier words can provide **keys** for any later word.
- All-to-all connections otherwise—need lots of tarined weights and compute.
- Queries and keys project into a vector space of dimension $d \ll n$, in which dot-products again quantify the strength of association.

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- Queries and keys project into a vector space of dimension $d \ll n$, in which dot-products again quantify the strength of association.
- So we multiply the $T \times d$ **key matrix** K and the $d \times T$ **query matrix** Q.

Transformers and Attention—cont'd



Multiplying KQ creates "hot spots" of gratified attention. Composing that on value vectors V updates the LLM's implicit understanding of words in the *latent* space.

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Is it still "All You Need"? Doing insanely well with mathematical theorem-proving right now...

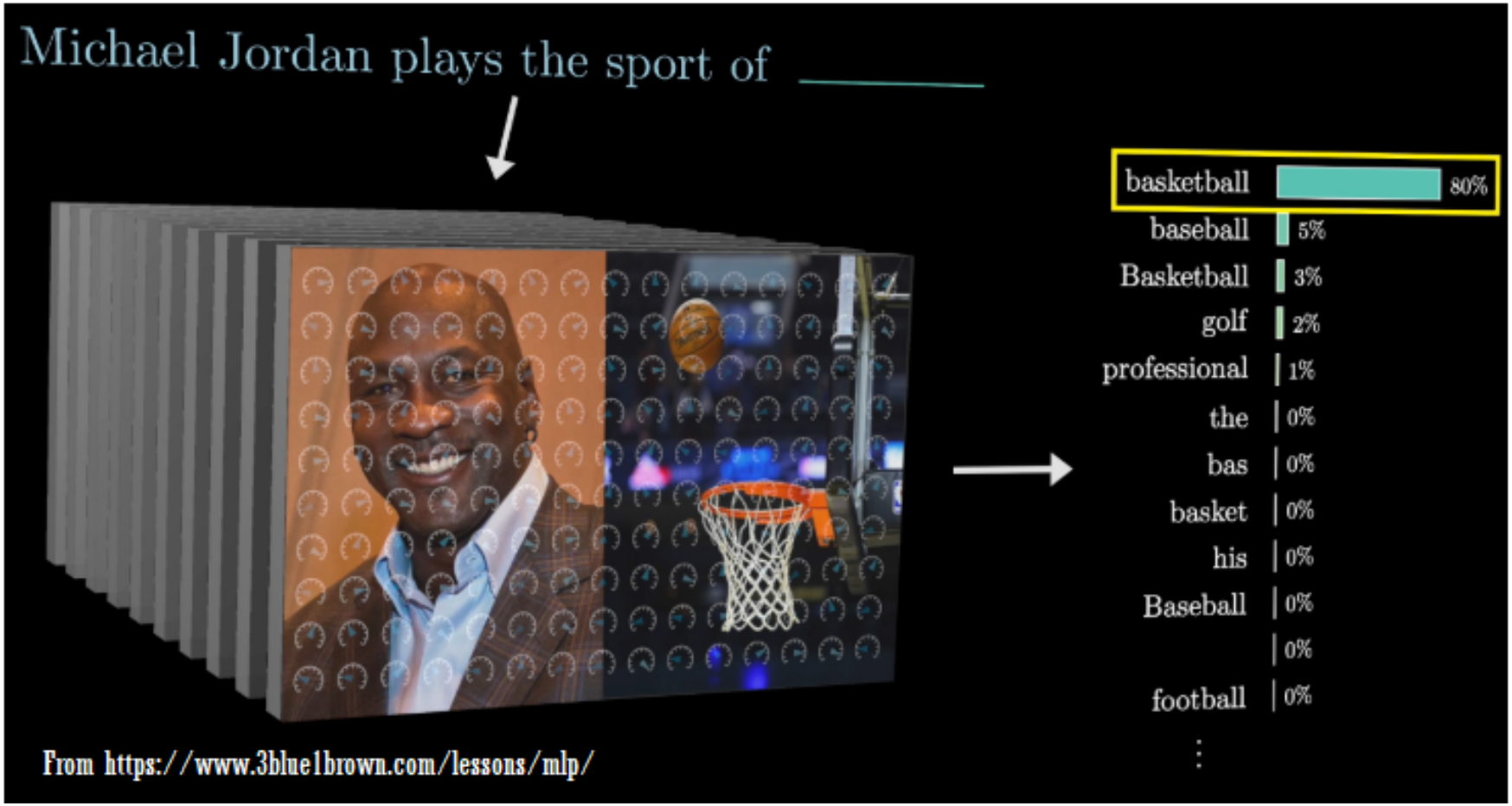
One Further Prediction Benefit

The transformer architecture simultaneously gives predictions for each word in the presented text based on all words before it.



Layers And Layers And Layers And Yet...

...only the outputs of the last (top) layer are used to give probabilities.



Has the network already extracted all useful info from its lower layers?

Extra: Why Token Values Are Called "Logits"

x_1	y_1
1	15.454
17	12.507
33	11.988
49	11.571
65	11.173
81	10.962
97	10.8
9,488 more rows Show all	
151921	−10.923
151934	−13.514

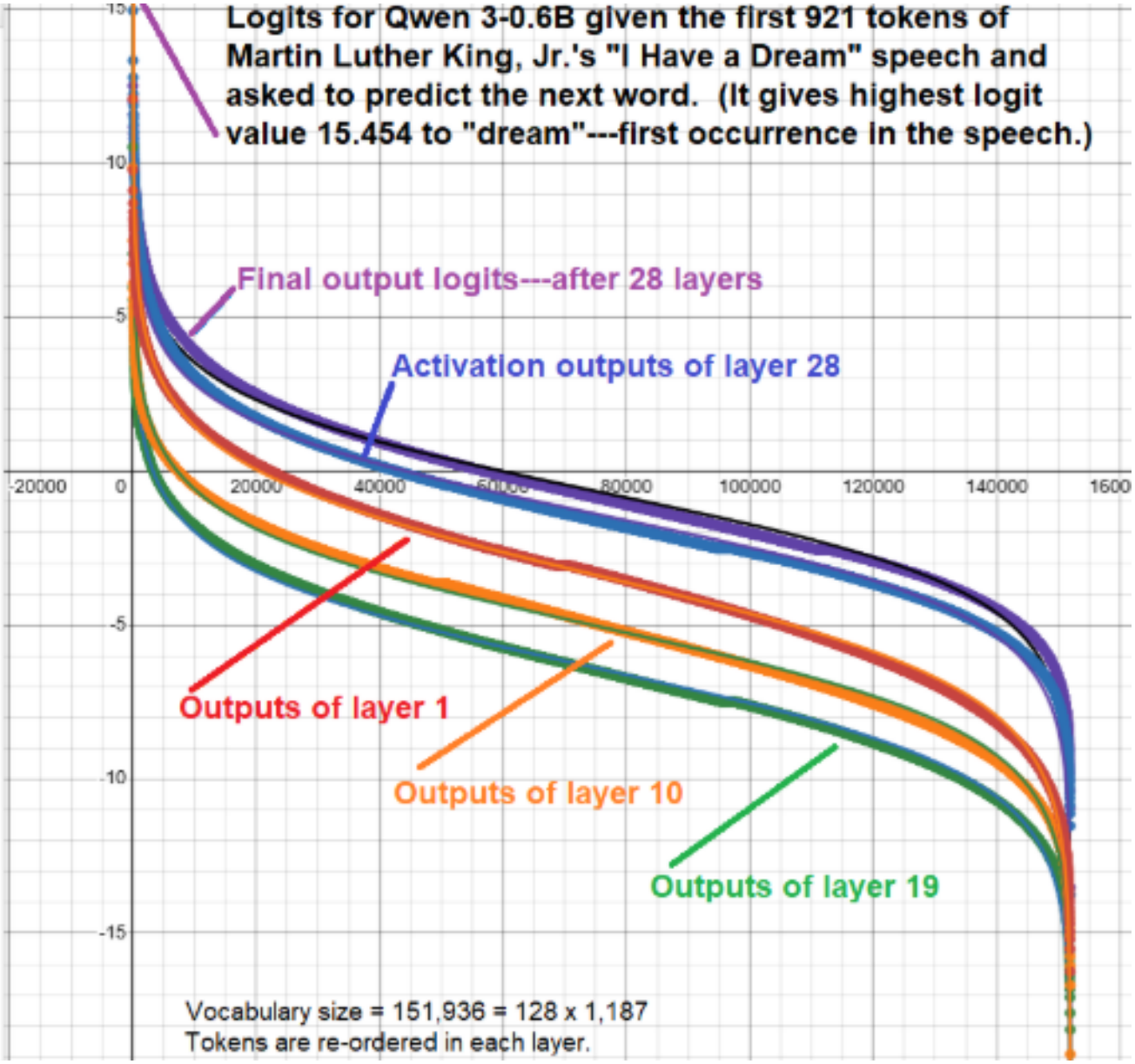
$$y_1 \sim a + b \ln\left(\frac{(151936 - x_1)}{(x_1)}\right)$$

REGRESSION PARAMETERS

$a = -0.68774$ $b = 1.58918$

STATISTICS $R^2 = 0.9884$ RESIDUALS e_1 plot

x_2	y_2



Steep At Top Makes Sense, But Why At Bottom?

1. `" dream"` = 15.454
2. `"strong"` = 14.774
3. `"heart"` = 14.454
4. `" heart"` = 14.409 (this duplication/spacing issue needs to be resolved too)
5. `"faith"` = 14.280
6. `" hope"` = 14.155 ...

Then things settle down so that by 200 or so tokens in, consecutive logit values differ only in the 0.001 place. But at the end:

151,931. `"Shu "` = -12.717
151,932. `"Die Dai "` = -12.871
151,933. `"Neng Hao "` = -13.330
151,934. `".EventType"` = -13.514
151,935. `"<|endoftext|>"` = -inf
151,936. `"<|im_end|>"` = -inf.

Granted, all the probabilities for these tokens obtained after softmax are microscopic. But why should the probability for "Die Dai " be $e^{0.459} = 1.58\ldots$ times higher than that for "Neng Hao "?