CSE396 Problem Set 7 Due 4/12, 11:59pm

Reading:

The **Second Prelim Exam** is fixed for **Thursday, April 26** in class period. It is technically "cumulative" because stuff from the first month may figure on the exam the way they have on Assignments 5 and 6. But it will focus on the domain of Assignments 5–8.

Tuesday's lecture will begin Chapter 3. The chapters get shorter now and the nature of the material breaks down into fewer categories after section 3.2. From section 3.2 we will hark back to PDAs in chapter 2, but not with the text's PDA notation.

Assignment 7, due Thu. 12 April, 11:59pm under the usual terms.

(1) This problem is "HW7 Online Part" on *TopHat*, worth 20 pts. as before. (The first seven questions were written by the TAs Jacob Ekstrum and Jim Dobler.)

(2) Let L be the language of balanced-parentheses strings that are mirror-image symmetric. For example, ()()() and ((())) belong to L but (())() does not. Does ((((((((())))))))))(((((((((()))))))))) belong to L? Use the latter as a hint in a CFL Pumping Lemma technique to prove that L is not a context-free language. (For a formal definition of L, let $\tilde{x}$ denote the result of flipping every '(' in x to ')' and vice-versa; then $L = \{ x : x \text{ is balanced and } x = \tilde{x}^R \}$. 18 pts.)

(3) Now define $L = \{a^i b^j a^k : j > i \wedge j > k\}$. Prove that L is not a CFL. Your proof may take some shortcuts but should show both "pumping up" and "pumping down." (24 pts.)

(4) Consider the following context-free grammar $G = (V, \{0, 1\}, R, S)$ with rules:

$$\begin{aligned} S &\rightarrow 0B \mid 1A \\ A &\rightarrow 0 \mid SA \mid B1B0 \\ B &\rightarrow 1 \mid S1S \mid AB1 \end{aligned}$$

Define E to be the language of binary strings x such that $\#1(x) - \#0(x)$ is a multiple of 3. Note that -3 and -6 count as multiples of 3.

(a) Prove that $L(G) \subseteq E$ using the structural induction technique from class. (18 pts.)

(b) Find a string z in which both $\#1(z)$ and $\#0(z)$ are positive and their difference is a positive multiple of 3 and yet $z \notin L(G)$. Use the 10th and last question on *TopHat* as a guide. (3 pts., for 21 on the problem and 83 on the set)