Define $A$ to be the set of strings $x \in \{a, b, c\}^*$ such that $x$ begins and ends with different letters.

(a) Does the empty string belong to $A$? Does any string of length 1?

(b) Design a DFA $M$ such that $L(M) = A$. For full credit you must have comments that explain the design strategy.

(c) Give a regular expression $r$ such that $L(r) = A$. You are not expected to apply the formal algorithm from the text and lectures but rather to “sight-read” $M$ and/or design $r$ from scratch. Either way you must give scratchwork and/or at least one sentence of explanation.
Let $N = (Q, \Sigma, \delta, s, F)$ be the NFA with $Q = \{1, 2, 3, 4\}$, $\Sigma = \{a, b\}$, $s = 1$, $F = \{4\}$, and $\delta$ given by the arcs $(1, \epsilon, 2)$, $(1, b, 2)$, $(1, b, 4)$, $(2, a, 3)$, $(3, a, 3)$, $(3, b, 2)$, $(3, \epsilon, 4)$, $(4, a, 1)$, and $(4, b, 2)$, shown by:

(a) Calculate a DFA $M$ such that $L(M) = L(N)$ (no “comments” needed if the method is clear).

(b) Find a string $x$ such that $N$ can process $x$ from 1 to any one of its four states—figuratively speaking, such that $x$ “lights up” all four states of $N$.

(c) Using your $x$ from part (b), find a string $y$ such that not only does $N$ fail to accept $xy$, but also for all $z \in \Sigma^*$, $N$ does not accept $xyz$. 
Multiple Choice.

Please circle the unique best answer or use the fill-in blanks at bottom. Justifications may help for partial credit but are not needed. The first three questions refer to the following DFA $M$:

1. Which of the following is true about the states of $M$?
   (a) States 1 and 3 are equivalent.
   (b) States 2 and 5 are equivalent.
   (c) States 5 and 6 are equivalent.
   (d) All pairs of states are inequivalent—this is a minimum DFA.

2. Let $r = a \cup (a \cup b)^* (b \cup ba)$. In terms of $M$, the regular expression $r$ is:
   (a) correct, meaning $L(r) = L(M)$.
   (b) sound, meaning $L(r) \subseteq L(M)$, but not correct.
   (c) comprehensive, meaning $L(M) \subseteq L(r)$, but not correct.
   (d) Neither sound nor comprehensive.

3. The language of strings with three consecutive $a$’s in them is:
   (a) contained in the complement of $L(M)$.
   (b) equal to the complement of $L(M)$.
   (c) a subset of $L(M)$.
   (d) not a regular language.

4. The complement of a language $A \subseteq \Sigma^*$ always equals:
   (a) $\emptyset$.
   (b) $A \cap \Sigma^*$.
   (c) $A \setminus \Sigma^*$.
   (d) $\Sigma^* \setminus A$.

5. Suppose $G$ is a directed graph whose edge relation is transitive and there is a path from node $u$ to a different node $v$ in $G$. Then what do we know about $G$?
   (a) There is a path from $v$ back to $u$.
   (b) There is an edge from $u$ to $v$.
   (c) $G$ has a self-loop at $v$.
   (d) $G$ is really an undirected graph.

(last problem is overleaf→)
(4) (24 pts.)

Over $\Sigma = \{a, b, c\}$, define $L = \{ xcy : \#a(x) > \#b(y) \ (x, y \in \Sigma^*) \}$. Prove via the Myhill-Nerode technique that $L$ is not a regular language. End of Exam