CSE 396 Review Session Mon May 13, 2019 for 2019

\[ H = \exists \left< p \right>: \left< p, i \right> \text{ prints "Hello World"?} \]

Is \( \therefore \): because there is just one computation and the print action, if it occurs, is detectable by a modified Java interpreter, so recognizable.

Not decidable: show \( A_m \leq_m H \). We need to map

\[ (M, w) \mapsto \text{p s.t. } (M, w) \in A_m \quad \Rightarrow \quad \text{P(e) writes } "H" \quad \text{(it might not halt but we don't care)} \]

\[ (M, w) \notin A_m \quad \Rightarrow \quad \text{P(e) never writes } "H" \]

Simulate \( M(w) \) using \( \text{Thm, which never writes } "H" \)

If \( M \) is a when accept

printIn ("Hello World")

\( \text{executed if and only if } M \text{ accepts } w. \)

Word to the wise: For problem about two machines, try fixing one then the other.

The map \( f \) is computable because it merely drops \( M \) and \( w \) into a pre-coded routine.

\( L = \{ <M, M> : A_m \leq_m L(M) \} \)

\( L(M) = \Sigma^* \quad \text{for } f(M) = < M_0, M > \)

\( L(M) = \Sigma^* \quad \text{for } \text{accept } L(M) \quad \text{and } L(M_0) = \Sigma^* \)

\( L = \{ <M, M_2> : A_m \leq_m L(M_2) \} \quad \text{for } \text{all } M, M_2 \in \Sigma^* \)

This \( f \) is computable in \( O(m^2) \).

For basic universal computation is (e.g. but) unrecognizable

\( c/05 \)

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\( c/05 \)
L_2 = \{ \langle M \rangle : \forall w \ M(w) \uparrow \}  \\
L_2 = \{ \langle M \rangle : \exists w \ M(w) \uparrow \}  \\
L_2 = \{ \langle M \rangle : \exists w \ M(w) \downarrow \}  \\
\text{Under the identification of Halting with } \text{All}  \\
L_2 \text{ is essentially } \text{NEFM}, \text{ which is c.e.}  \\
\text{So } L_2 \text{ is c.e. And undecidable since } \text{NEFM is undecidable.}  \\
L_3 = \{ \langle X, M \rangle : \text{something about } M \text{ that} \} = \emptyset  \\
L_4 \text{ on TM Hat } = \{ \langle M, L \rangle : L(M) \geq 20 \}  \\
L_1 \text{ on 2018 Prod } = \{ \langle M \rangle : L(M) \geq 1 \}  \\
\text{Like AllM because all in while langs have 1-1 correspondence}  \\
\text{Run M(1)}  \\
\text{Accepts}  \\
\text{Rejects}  \\
\text{X has a } T, \text{ reject}  \\
\text{X is even, so compute } N: x + 2 = 0  \\
\text{Compute } y = \text{the string of } n \text{ in binary}  \\
L_5 = \{ \langle M \rangle : L(M) \geq 1 \}  \\
L_5 = \{ \langle M \rangle : L(M) \geq 0 \}  \\
\text{Is c.e.}  \\
\text{Is undecidable via All or } \text{N. With Smith! }  \\
\text{Difficult. } L(M) \geq 5 \Rightarrow L(M) \geq 4 \Rightarrow L(M) \geq 2 \Rightarrow L(M) \geq 1 \Rightarrow L(M) \geq 0 \Rightarrow L(M) \geq 0 \Rightarrow L(M) \geq 0 \Rightarrow \{ \text{true} \}  \\
\text{A.M. } = m L_5  \\
\text{L}_1 = \{ \langle X, Y \rangle = Y = X^2 : \text{either } X \text{ begins with } 0 \text{ and } Y = X^R \}  \\
\text{Or } X \text{ does not begin with } 0 \text{ and } Y = X^R  \\
\text{If } X \text{ begins with } 1 \text{, sim as PDA for } S  \\
S \rightarrow OS \text{ if } S \text{ is a signal for } OS \rightarrow OS \rightarrow O  \\
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(1) (50 pts.)

Classify each of the following languages \(L_1, \ldots, L_{10}\) over \(\Sigma = \{0,1\}\) according to whether it is

(a) regular;
(b) a DCFL but not regular;
(c) a CFL but not a DCFL;
(d) in \(\text{P}\) but not a CFL;
(e) decidable but definitely not in \(\text{P}\) or at least \(\text{NP}\)-hard;
(f) c.e. but not decidable;
(g) co-c.e. but not c.e.; or
(h) neither c.e. nor co-c.e.

You need not justify your answers, but brief justifications may help for partial credit. With \(r = 1^*01*01^*\), the predicates for \(L_1\) through \(L_4\) are equivalent to, respectively, \(L(M) \supseteq L(r),\) \(L(M) \subseteq L(r),\) \(L(M) = L(r),\) and \(L(M) \cap L(r) \neq \emptyset.\) Strings in \(L_7\) have length a multiple of 3 and are indexed 1, 3, 5, 3, 7, \ldots for some \(m > 0.\)

1. \(L_1 = \{ \langle M \rangle : M \text{ accepts all strings that have exactly two 0s in them} \}.
2. \(L_2 = \{ \langle M \rangle : \text{ all strings that } M \text{ accepts have exactly two 0s in them} \}.
3. \(L_3 = \{ \langle M \rangle : M \text{ accepts exactly the strings that have exactly two 0s in them} \}.
4. \(L_4 = \{ \langle M \rangle : M \text{ accepts some string that has exactly two 0s in it} \}.
5. \(L_5 = \) the set of strings that have exactly two 0s in them.
6. \(L_6 = \) the set of even-length strings that have exactly two 0s, and they are in the two center positions.
7. \(L_7 = \) the set of strings \(x\) with exactly two 0s and they are in the places indexed \(m\) and \(2m.
8. \(L_8 = \) the complement of \(L_7.
9. \(L_9 = \{ \langle N, x \rangle : N \text{ is an NFA and } N \text{ accepts } x \}.
10. \(L_{10} = \{ \langle N, x \rangle : N \text{ is an NFA and } N \text{ accepts all strings of the same length as } x \}.

P11. (g) or "11. (g)" or (safest) "11. \(L_{11}\) is (g) r.e. but undecidable."

Answers: