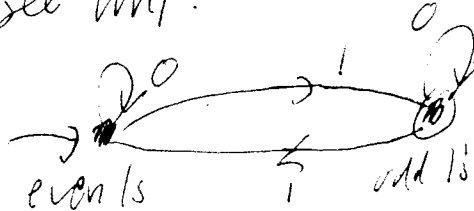


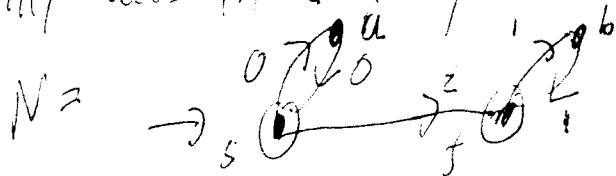
$L_1 = \{x \in \{0,1\}^* : x \text{ has an odd number of } 1\text{'s}\}$. How to design a regular expression r_1 such that $L_1 = L(r_1)$?

Directly: $L_0 = \{x : x \text{ has an even number of } 1\text{'s}\} (= \bar{L}_1)$ has the regular expression $r_0 = (0^*10^*10^*)^*$ (with two levels of nesting).
 So take $r_1 = r_0 \cdot 0^*10^*$ since adding one 1 to "even" makes "odd".
 This gives $(0^*10^*10^*)^*0^*10^*$, which simplifies to $(0^*10^*10^*)10^*$ or to $(0^*10^*1)0^*10^*$ - see why?

By Machine: Design $M_1 =$  so $L(M_1) = L_1$.

By the formula (to be) given in class, $L(M_1) = (0 + 10^*1)^*10^*$
 Also $0^*1(0 + 10^*1)^*$. How can all these different expressions be equally correct? We'll come to this in the last weeks on complexity.

Example 2: Build machines for $r = (00)^*(11)^*$. The NFA really needs an ϵ -arc for the concatenation but not elsewhere:

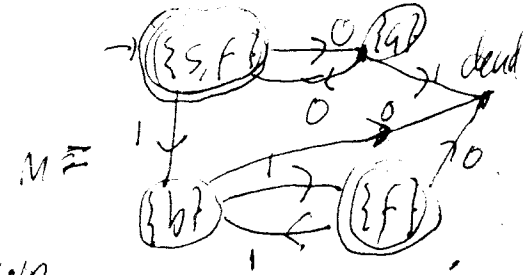


q :	0	1
s	a	f
a	s, f	s
b	\emptyset	f
f	\emptyset	b

$$L(s, 0) = \underline{\epsilon}(s, 0) \cup \underline{\epsilon}(f, 0) = \{a\} \cup \emptyset = \{a\}$$

$$L(s, 1) = \emptyset \cup \{b\} = \{b\}$$

We didn't need to make the start state accepting - it has an ϵ -arc to the accepting state f. But what we need to know is $S = \{s, f\}$ for the DFA M :



END