Name and St.ID#:_____

CSE396, Spr'19 Second Prelim Exam Apr. 25, 2019

Closed books and laptops, one notes sheet allowed, closed neighbors, 75 minutes. Do ALL THREE problems **on these exam sheets**. Extra sheet(s) may be requested if needed. Please *show all your work*—this may help for partial credit. The exam totals 80 pts., subdivided as shown.

In problem (1), you may assume the alphabet Σ is $\{a, b\}$, and the *difference* of two languages $A, B \subseteq \Sigma^*$ means $A \setminus B$, which equals $A \cap \tilde{B}$.

(1) ($4 \times 3 = 12$ pts.) *Multiple Choice.*

Please circle the unique best answer or use the fill-in blanks at bottom. Justifications may help for partial credit but are not needed.

1. The class of CFLs is closed under
   (a) Union.
   (b) Intersection.
   (c) Complementation.
   (d) Difference.
2. The class of DCFLs is closed under
   (a) Union.
   (b) Intersection.
   (c) Complementation.
   (d) Difference.
3. Given DFAs M_1, M_2 and a CFG G, which problem is *not* decidable?
   (a) Whether $L(M_1) = \emptyset$.
   (b) Whether $L(M_1) = L(M_2)$.
   (c) Whether $L(M_1) = L(G)$.
   (d) Whether $L(G) \neq \emptyset$.
4. If L is undecidable, then its complement $\tilde{L}$ could be:
   (a) Regular
   (b) Context-free.
   (c) Decidable.
   (d) Recognizable (i.e., c.e.).

1. _____ 2. _____ 3. _____ 4. _____

(2) (2 + 6 + 18 + 6 = 32 pts.)

Define E to be the language of strings over $\Sigma = \{a, b\}$ that begin with b and in which every nonempty "block" of a's had odd length. "Block" means a *maximal* substring of a's, and another way of saying this is that between every b in the string and the next b (or the end of the string), the number of a's is either zero or odd. Examples of strings in E are *babba* and *babaaab*, but not *babbaa* or *babaab*. Define G to be the following context-free grammar: overleaf.

$$S \to bB \mid bAS \qquad A \to BSb \mid AA \mid ab \qquad B \to a \mid AB \mid BaaS$$

(a) Is $\epsilon \in E$? _____ What about the string b^i, for any $i \geq 1$? _____

(b) Find an ambiguous string in $L(G)$ and show two different parse trees for it.

(c) Prove by the structural induction technique that $L(G) \subseteq E$.

(d) Suppose we add the rule $S \to b$. Show that this makes the new grammar G' *unsound* for E by giving a *leftmost* derivation in G' of a string $x \notin E$.

(3) (12 + 12 + 12 = 36 pts.) (Last problem on the exam)

Define $L_1 = \{a^i b^j c^k : j \leq i,\ i, j, k \geq 1\}$ and $L_2 = \{a^i b^j c^k : j \leq k,\ i, j, k \geq 1\}$.

(a) Design a CFG G such that $L(G) = L_1 \cup L_2$.

(b) Prove that $L_1 \cap L_2$ is not a CFL.

(c) Sketch in prose a 2-tape TM M such that $L(M) = L_1 \cap L_2$. You must give enough detail about character-level operations to indicate where and why your M is not a pushdown automaton.