CSE396, Spring 2021 Problem Set 7 Due Tue. 4/6, 11:59pm

Reading: Please read the rest of Chapter 3 for next week, and also section 4.1 on basic algorithms. Tuesday's lecture will include multi-tape Turing machines and will define push-down automata as a special case of 2-tape TMs, thus harking back to section 2.2. The "Notes on Turing Machines and PDAs" on the course webpage have screenshots of machines being demonstrated, and also a handwritten sketch of what the text talks about at the end of chapter 3. The thrust of this is that once we see how Turing machines can do the operations needed to build an assembly-level interpreter, we can specify them via higher-level pseudocode as the text does in chapter 4, rather than arc-node detail.

Homework—part online (TopHat), part written, and all *individual work*:

(1) Using *TopHat*, the "Worksheet" titled *S21 HW7 Online Part* (10 Qs, 20 pts.)

The other two problems are to be submitted as PDFs using the *CSE Autograder* system.

(2) Over $\Sigma = \{a, b, c\}$, consider the following context-free grammar $G = (V, \Sigma, \mathcal{R}, S)$ with rules:

$$S \to BAc \mid SB \mid bc, \quad A \to cBA \mid Bcb, \quad B \to cS \mid a$$

Let T be the language of strings x such that between any two b's in x there are at least two c's.

(a) Prove that $L(G) \subseteq T$ using the structural induction technique from lectures and notes. All variables will include the property that all strings they derive by themselves belong to T but you will need to add further properties that at least one of the variables must uphold. Defining effective properties P_S, P_A, P_B that are upheld by all the rules counts for 9 pts. and verifying each non-terminal rules is 3 pts., for 24 total.

(b) Suppose we change the first rule for B from $B \to cS$ to $B \to aS$. Demonstrate (by derivation or parse tree) a string in the new grammar that does not belong to T. (6 pts., for 30 total)

(3) Prove via the CFL Pumping Lemma that the following two languages are not CFLs. Both are over the same alphabet $\Sigma = \{a, b, c\}$.

(a) $L_a = \{ucv : \#a(u) = \#a(v) \ \wedge\ \#b(u) = \#b(v)\}$.

(b) $L_b = \{a^i b^j a^k : i < j \ \wedge\ k < j\}$.

As a hint for L_a, consider test strings of the form $a^N b^N c a^N b^N$. For L_b, your proof may take some shortcuts but should show both pumping up and pumping down. (12+18 = 30 pts., for 80 total on the set)