CSE396, Spr'19 First Prelim Exam Mar. 12, 2019

Sample exam for CSE396, Spring 2021. Content similar but your format will differ.

Closed books and laptops, one notes sheet allowed, closed neighbors, 75 minutes. Do ALL FOUR problems **on these exam sheets**. Please *show all your work*—this may help for partial credit. The exam totals 80 pts., subdivided as shown.

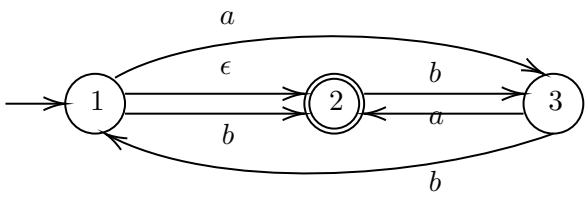
Notation: All problems on this exam use alphabet $\Sigma = \{ a, b \}$. $\#c(x)$ stands for the number of occurrences of the character c in the string x.

(1) (20 pts.)

Let $A = \{ x \in \{ a, b \}^* : bb$ is a substring of x and $\#a(x)$ is even $\}$. Design a deterministic finite automaton (DFA) M such that $L(M) = A$. A node-arc diagram that shows the start and final states clearly is good enough—you need not write out tables or "$M = (Q, \Sigma, \delta, s, F) \ldots$" etc. For full credit, you must either have comments explaining a design strategy that makes the correctness of your M clear, or you must use theorems to build M from smaller machines. Trial-and-error may take too long.

(2) (15 + 12 = 27 pts.)

Let N be the NFA defined by $N = (Q, \Sigma, \delta, s, F)$ with $Q = \{1, 2, 3\}$, $\Sigma = \{a, b\}$, $s = 1$, $F = \{2\}$, and δ given by the arcs $(1, b, 2)$, $(1, \epsilon, 2)$, $(1, a, 3)$, $(2, b, 3)$, $(3, b, 1)$, and $(3, a, 2)$ as shown in the following node-arc diagram:



(a) Calculate a DFA M such that $L(M) = L(N)$.

(b) Calculate a regular expression r such that $L(N) = L(r)$.

(3) (5 × 3 = 15 pts.) *True/False.*

Please write out the words `true` and/or `false` in full. No justifications are needed.

(a) If $A^* = A$, then the language A includes the empty string.

(b) If A and B are regular languages recognized by 3-state DFAs, then $A \cap B$ can be recognized by a 6-state DFA.

(c) The empty relation on a nonempty set is transitive.

(d) The intersection of two non-regular languages is always non-regular.

(e) If there is a string w such that no string x in a regular language A has w as a substring, then every DFA M such that $L(M) = A$ has a dead state.

(a) _____ (b) _____ (c) _____ (d) _____ (e) _____

(4) (18 pts.)

Define $L = \{x \in \{a, b\}^* : |x| \text{ is even and there is a } b \text{ in the left half of } x\}$. For instance, ba, $aabaaa$, and $baaaaa$ belong to L, but ϵ, ab, and $baaaa$ do not—the last fails because its length is odd. Prove using the Myhill-Nerode technique that L is not a regular language.