

*Your Prelim II will have the same terms as your Prelim I did:* One notes sheet and text recommended, notes OK, Internet for connectivity only, closed neighbors, 70 minutes. Do ALL THREE problems in a format that can be exported to PDF for submission via *CSE Autolab* using the same logistics as homeworks. Please *show all your work*—this may help for partial credit. The exam totals 80 pts., subdivided as shown.

*Extra notes that were on the Spr'19 exam:* In problem (1), you may assume the alphabet  $\Sigma$  is  $\{a, b\}$ , and the *difference* of two languages  $A, B \subseteq \Sigma^*$  means  $A \setminus B$ , which equals  $A \cap \tilde{B}$ .

**(1) ( $4 \times 3 = 12$  pts.)** *Multiple Choice.*

Please circle the unique best answer or use the fill-in blanks at bottom. Justifications may help for partial credit but are not needed.

1. The class of CFLs is closed under
  - (a) Union.
  - (b) Intersection.
  - (c) Complementation.
  - (d) Difference.
2. The class of DCFLs is closed under
  - (a) Union.
  - (b) Intersection.
  - (c) Complementation.
  - (d) Difference.
3. Given DFAs  $M_1, M_2$  and a CFG  $G$ , which problem is *not* decidable?
  - (a) Whether  $L(M_1) = \emptyset$ .
  - (b) Whether  $L(M_1) = L(M_2)$ .
  - (c) Whether  $L(M_1) = L(G)$ .
  - (d) Whether  $L(G) \neq \emptyset$ .
4. If  $L$  is undecidable, then its complement  $\tilde{L}$  could be:
  - (a) Regular
  - (b) Context-free.
  - (c) Decidable.
  - (d) Recognizable (i.e., c.e.).

**(2) (2 + 6 + 18 + 6 = 32 pts.)**

Define  $E$  to be the language of strings over  $\Sigma = \{a, b\}$  that begin with  $b$  and in which every nonempty “block” of  $a$ ’s had odd length. “Block” means a *maximal* substring of  $a$ ’s, and another way of saying this is that between every  $b$  in the string and the next  $b$  (or the end of the string), the number of  $a$ ’s is either zero or odd. Examples of strings in  $E$  are  $babba$  and  $babaaab$ , but not  $babbaa$  or  $babaab$ . Define  $G$  to be the following context-free grammar: overleaf.

$$S \rightarrow bB \mid bAS \quad A \rightarrow BSb \mid AA \mid ab \quad B \rightarrow a \mid AB \mid BaaS$$

- (a) Is  $\epsilon \in E$ ? ..... What about the string  $b^i$ , for any  $i \geq 1$ ? .....
- (b) Find an ambiguous string in  $L(G)$  and show two different parse trees for it.
- (c) Prove by the structural induction technique that  $L(G) \subseteq E$ .
- (d) Suppose we add the rule  $S \rightarrow b$ . Show that this makes the new grammar  $G'$  *unsound* for  $E$  by giving a *leftmost* derivation in  $G'$  of a string  $x \notin E$ .

**(3) (12 + 12 + 12 = 36 pts.)** (Last problem on the exam)

Define  $L_1 = \{a^i b^j c^k : j \leq i, i, j, k \geq 1\}$  and  $L_2 = \{a^i b^j c^k : j \leq k, i, j, k \geq 1\}$ .

- (a) Design a CFG  $G$  such that  $L(G) = L_1 \cup L_2$ .
- (b) Prove that  $L_1 \cap L_2$  is not a CFL.
- (c) Sketch in prose a 2-tape TM  $M$  such that  $L(M) = L_1 \cap L_2$ . You must give enough detail about character-level operations to indicate where and why your  $M$  is not a pushdown automaton.

END OF EXAM.