# CSE396, Spr'21 Second Prelim Exam Apr. 20, 2021

One notes sheet and text recommended, notes OK, Internet for connectivity only, closed neighbors, 70 minutes. Do ALL THREE problems in a format that can be exported to PDF for submission via *CSE Autolab* using the same logistics as homeworks. Note that the last problem has a **choice**: you must attempt EXACTLY ONE of (3a) XOR (3b) and indicate clearly which one you are trying. Please *show all your work*—this may help for partial credit. The exam totals 80 pts., subdivided as shown. All problems on this exam use alphabet $\Sigma = \{ a, b \}$.

**(1) (6 + 5 + 21 + 6 + 6 = 44 pts.)**

Consider the language $E = \{x \in \{a, b\}^* : bb \text{ is not a substring of } x\}$. That is, strings in $E$ do not have two consecutive $b$'s. Let $G$ be the context-free grammar with rules:

$$\begin{aligned} S &\rightarrow AbS \mid Ab \\ A &\rightarrow a \mid SA \mid AB \\ B &\rightarrow baSa \mid BB \end{aligned}$$

(a) Give two different parse trees for the string $abab$. (6 pts.)

(b) Give a leftmost derivation for a string $x \in L(G)$ that has a substring $aa$. (5 pts.)

(c) Prove that $L(G) \subseteq E$ via the structural induction technique. Note that all three variables must derive strings in $E$ because the property of not having a $bb$ carries through to substrings. You must add stronger properties to your $P_S$, $P_A$, and $P_B$ to make the induction go through. The grading is 9 pts. for those properties and 12 pts. for handling the rules.

(d) Can $S$ derive some string that ends in $a$? Justify your answer. (6 pts.)

(e) What would go wrong if we shortened the rule $B \rightarrow baSa$ to become $B \rightarrow baS$ to make a different grammar $G'$? Show how $G'$ would become unsound with regard to $E$ (your choice of parse tree, derivation, or clear enough prose answer). (6 pts.)

**(2) (18 pts.)**

Recall the definition of "block" in a string as being a maximal nonempty substring of the same character. For example, the string $x = aabbaaab$ has one block of two $a$'s and one block of three $a$'s, but we don't count the zero $a$'s between the two consecutive $b$'s or the end of the string as a block of $a$'s. The string $x$ does have a block of two $b$'s and one block of a single $b$ at the end. Now define

$$L = \{x \in \{a, b\}^* : \text{all blocks of } a\text{'s in } x \text{ have the same length}\}.$$

Prove via the CFL Pumping Lemma that $L$ is not a context-free language. Note that $x$ does not belong to $L$ but the string $x' = aaabaaabbaaa$ does belong to $L$.

**(3) (18 pts. total)**

This is a **choice** of problems, one referring to the language $E$ in problem (1) (*not to the grammar there*), the other to $L$ in problem (2). *The problems are on the next page.*

**(3a)** Design a context-free grammar $G'$ such that $L(G')$ equals the language $E$ in problem (1). Because $E$ is in fact a regular language, here is a possible "machine-based" recipe as a hint: Design a DFA $M$ such that $L(M) = E$. Allocate a variable for each state, and make rules that imitate instructions of $M$, including the possibility of stopping in an accepting state when the input string $x$ comes to an end. If you design your $G'$ by other means, you must have a comment explaining why your $G'$ is correct.

XOR

**(3b)** Design a 2-tape TM $M$ such that $L(M)$ equals the language $L$ in problem 2. You may either draw $M$ on paper (with some comments) or you may give a detailed prose sketch. If you give a prose sketch, it must be detailed enough say:

- whether $M$ ever moves its input tape head left; and
- what the second tape does and whether it obeys the rules of a pushdown automaton—saying where not if not.