Recitation Notes for Week 2, Spring 2010 CSE 396

What is 0^0? Why do people say $0^0 = 1$?

The reason we care: For any language A over an alphabet Σ, and $i \in \mathbb{N}$, define $A^i = \left\{\begin{array}{l}\text{concatenations of } i \text{ strings,}\\ \text{each string belonging to } A\end{array}\right\}$.

For instance, if $A = \{ab, aba, ba\}$ and $i = 2$, then

$A^2 = \{abab, ababa, abba, abaab, abaaba, baab, baaba, baba\}$

- Note that we did not list "ababa" twice, even though it came up twice, as $ab \cdot aba$ and later as $aba \cdot ba$. This is because A^2 is a set, not a list.
- Note that the strings being concatenated don't have to be different: we included $ab \cdot ab$, $aba \cdot aba$, and $ba \cdot ba$.
- Note that $A^2 \neq \{xx : x \in A\}$. The latter is just $\{abab, abaaba, baba\}$.
- This is a case of the more general definition of concatenation of languages which will be given (next week) in lectures. For languages A and B, (Jan 25 or 27)

$$\underline{A \cdot B} = \{xy : x \in A \ \& \ y \in B\}.$$

Then A^2 equals $A \cdot A$, ie the case $B = A$ here. This accords with how powering relates to multiplication in numerical math, but now with strings and symbols. How far does the analogy go?

Well, $A \cdot \emptyset = \{xy : x \in A \ \& \ y \in \emptyset\} = \{xy : \text{FALSE}\} = \emptyset$. Like $A \cdot 0 = 0$

$A \cdot \{\epsilon\} = \{xy : x \in A \ \& \ y \in \{\epsilon\}\} = \{xy : x \in A \ \& \ y = \epsilon\}$

$= \{x \cdot \epsilon : x \in A\} = \{x : x \in A\} = A$. Like $A \cdot 1 = 1$.