CSE 396 First Lecture Tue. Jan. 30 Spring 2018

# Topic: The Mathematics of Strings

**Q:** When is a decimal integer a multiple of 3?

x = 1056247

add digits 25

add digits 7

Fact: Can add the digits. The sum is a multiple of 3 iff the original number is.
"if and only if"

Base case: Not 0, 3, 6, or 9
so answer is no.

x = 1,100,000

add digits 2

Making x bigger did not make the digit sum bigger.

So "philosophically" the digit sum is not like in calculus:

> Tiny changes to the number can affect the digit sum majorly in either direction

1,099,999

46

10

10000000 1

45

9 ✓ yes

Note that technically the algorithm involves recursion, with single digits as base cases.

11011 base 2

× 11

11011 ← carry bits

11011

1010001

= 81 base 10

Q: How to write concatenation symbolically?

Let $X =$ "101" and $Y =$ "110"

Consider $Z =$ "101110". Should we write

$Z = X + Y$ or $Z = X \cdot Y$ ?

or use a completely different symbol, eg $Z = X \oplus Y$ ?

NB: $Z = X \# Y$ intends '#' as an actual char.
NB = nota bene, "note well"

Consider $W_2 = 101101 = XX$

$X^2$ //

$X^3 = W_3 = 101101101 \overset{?}{=} X+X+X$ or $X \cdot X \cdot X$

$X^4 = W_4 = 101101101101 \overset{?}{=} X+X+X+X$ or $XXXX$?

Consider $Z' = 110101 \neq Z = 101110$

Write $Z' = Y + X$? or $Z' = Y \cdot X$?

$X^1 = X$ $W_1 = X = 101$

$X^0 \overset{\text{def}}{=} \varepsilon$ $W_0 =$ "": empty something?

$\emptyset$: empty set

$\varepsilon$: empty string

Greek epsilon

Power Rule for Strings: $X^a \cdot X^b = X^{a+b}$ ($\lambda$ lambda also used.)

Hence desirable to think of string concat as a kind of multiply

Thru next week: bump up to sets of strings, called languages.