CSE 396 Lecture 1/31/17 Spring 2017 (1)

"The Mathematics of Strings". E.g.:

Is the number 125,832 divisible by 3?

Long division can be treated as a symbolic process

"Semi-Symbolic": Try adding up the digits

$1+2+5+8+3+2 = 21$

Theorem: x is divisible by 3 iff the sum of its digits is divisible by 3

Define: $L_3 = \{$ strings x of digits (numbers x given by): the number x is divisible by 3 $\}$.

New Stmt of Theorem: $x \in L_3 \leftrightarrow$ the sum of its digits is in L_3.

Try $12583\cancel{3}8$ sum $= 2\cancel{4}7 \to \cancel{6}\ 9$

Inductive Understanding:

Basis:
- If x = "0", "3", "6" or "9", say accept: $x \in L_3$.
- If x = "1", "2", "4", "5", "7", "8", say reject: $x \notin L_3$.

Induction: $x \in L_3$:

def Mul3(x):

$$\boxed{\begin{array}{l} y = \text{sum\_of\_digits}(x) \\ \text{return Mul3}(y) \end{array}}$$

Defn: An *alphabet* is a finite set of symbols. E.g.

DIG = '0', '1', '2', ---, '9'

ALPH = 'A', 'B', -, 'Z' $\quad |ALPH| = 26$

ASCII: includes a, --, z, includes DIG, and punctuation chars and control codes and more: $|ASCII| = 256$

Binary alphabet $B = \{'0', '1'\}$ or $\{'a', 'b'\}$

The letter Σ (capital Sigma) will stand for any alphabet.

Main Operation: $\bullet$ Concatenation.

"pom" $\bullet$ "p" = "pomp". Not commutative:
"p" $\bullet$ "pom" = "ppom" $\neq$ "pomp".

Defn: A *string* X *over* an alphabet Σ is a concatenation of 0 or more chars in Σ.

The *empty string*, denoted by ε or "", is a string.

The set of all *finite* strings over Σ is denoted by Σ^*. The $*$ means "zero or more but finite."

Relations: Given strings X, Y over Σ (say Σ = ALPH)

define $X \sim Y$ if $X \cdot Y$ is a repeated word, i.e. there is another string Z such that

$$X \cdot Y = Z \cdot Z$$

Is "POMP" ~ "OM"? yes because

$$POMP \cdot OM = \underset{Z}{POM} \cdot \underset{Z}{POM}$$

(in the definition)

Is the relation ~ reflexive? Yes: $X \cdot X = X \cdot X$

Is ~ symmetric, i.e. if $X \sim Y$, must $Y \sim X$?

Is OM ~ POMP? yes: = OMP.OMP

But how to prove this in general?

Is ~ transitive?

Added after lecture:

I.e. if $X \sim Y$ and $Y \sim W$, do we always get $X \sim W$?

No. Counterexample: X = POMP, Y = OM, W = YOMY

XY = POMPOM = POM.POM

YW = OM.YOMY = OMY.OMY. But

XW = "POMPYOMY" which is not a double-word.

You Cant Argue With a Counterexample!

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