CSE 396 Lecture 2/2/17 Spr 2017 ①

[First 35 minutes was a demo of the Turing Kit software, "Dragonsh" DFA.]

Picking up, I defined the relation $\sim$ on an alphabet Σ, actually on strings in $\underline{\Sigma^*}$, by:

$x \sim y$ if and only if there is a string u such that

$$x \cdot y = u \cdot u$$

"$\sim$ is transitive" means that for all x, y, and z: if $x \sim y$ and $y \sim z$ then $x \sim z$

Example: $x =$ POMP, $y =$ OM, $z =$ YOMY

$xy =$ POMPOM, $u =$ POM

$x' =$ OM, $y' =$ YOMY

$x' \cdot y' =$ OMYOMY, $u =$ OMY

$\therefore x \sim y$ and $y \sim z$, but $xz =$ POMP $\cdot$ YOMY there is no good u

$\therefore x \not\sim z$ so $\sim$ is not transitive.

Let's now say $X \approx Y$ if there exists a Z such that $X \cdot Z$ and $Y \cdot Z$ are both double words, I.e. both belong to the set

$$D = \{W : \text{there is a } u \text{ such that } W = u \cdot u\}.$$

Example: $X = POMP$, $Y = OM$. $Z = OM$ so $X \approx Y$: $XZ = POMPOM$, $YZ = OMOM$

How about $X' = POMPA$? no Z shorter than POMPA can make $XZ \in D$

or $X' = POMPO$? $Z = M$ works

$Y' = M$ $Z = M$ works here too.

$XZ \in D$ $W = MM$ so $X' \approx Y'$.

$W = TOMT$

Claim: • $\approx$ is reflexive too: $X \approx X$

because $Z = X$ makes $XX \in D$ twice over.

• Is $\approx$ symmetric? $\boxed{\text{yes because the def.}^n \text{ itself is "symmetric".}}$

• Is $\approx$ transitive? Try ~~W~~ $= M$? $W = MM$?

I.e. if $X \approx Y$ and $Y \approx W$, is $X \approx W$? no $W = TOMT$ breaks it

But, a reflexive and <u>symmetric</u> binary relation $R(x,y)$ does yield an <u>Undirected graph</u>. $G = (V, E)$

V = the set of items, here strings

$E = \{(x,y) : R(x,y)\}$.

<u>Convention</u>: Undirected graphs usually don't have or <u>omit</u> self-loops.

<u>Directed</u> graphs usually include them, as we saw for the DFA examples.

I.e. if $X \cdot Z$ and $Y \cdot Z$ are double words, must $X \cdot Y$ be one?

<u>Added</u>: <u>Challenge Question</u>. One can see that $X \sim Y \Rightarrow X \approx Y$ (take $Z = Y$). Does the <u>converse</u> always hold? ("Think ε")

[Extra notes done with a student after lecture]

We could define X is a double word by:

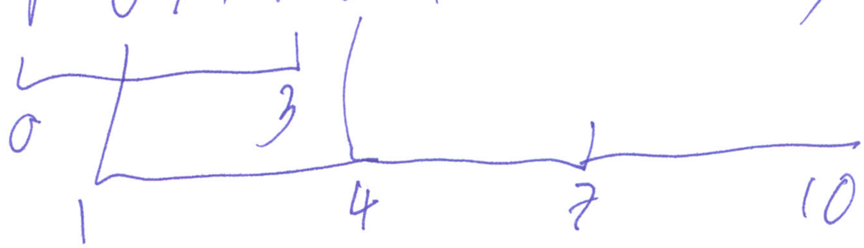
let $n = X$

n must be even, so $n = 2m$ for some m.

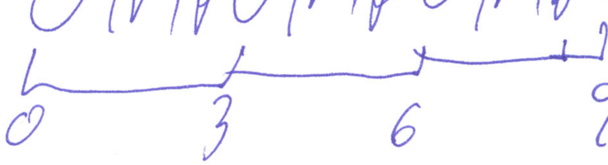
For all i, j such that $j = i + m$

the char X_i = the char X_j

POMPOM $n = 6,\ m = 3$



OMPOMPOMPO



Another way of saying this is that $\boxed{i \equiv j \bmod m} \Rightarrow \boxed{X_i = X_j}$ Once you see it's a congruence, it doesn't matter where you start in the cycle – it just keeps cycling.

So: $X \cdot Y$ is a double word

$\Longleftrightarrow$

$Y \cdot X$ is a double word.

Unexpected? But true.