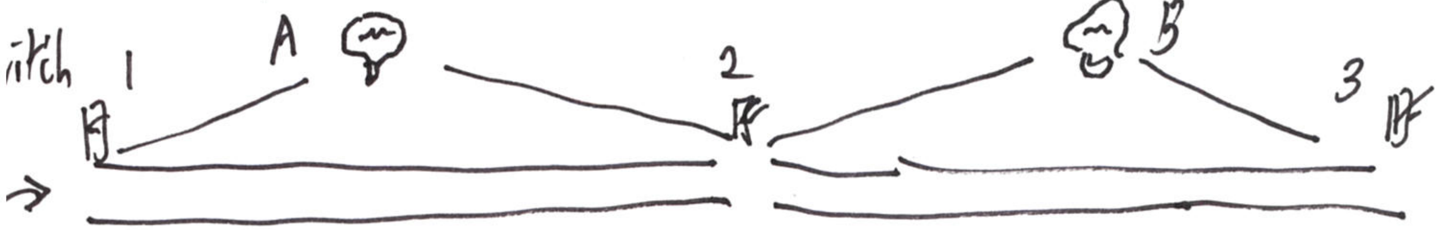
CSE 396 Lecture Thu 2/4 Spring 2016

<u>Assgt 1</u> will be posted this aft.

A "Real System" Example — Long Library Shelf Passage.



<u>Modeling</u>: Take $\Sigma = \{1,2,3\}$. Then a string $x \in \Sigma^*$ models a sequence of <u>0 or more</u> "flicks" of each of the three switches 1, 2, 3.

E.g. one left-to-right "pass" could be
$w \cancel{x} = 123$

Target Language $L = \{x \in \Sigma^*$: Assuming the lights were initially both off, executing x leaves them both off. $\}$

Is $w \in L$? yes: "123" $\in L$. In general, <u>how</u> can we tell?

- Above, L was defined by a <u>specification</u>: "Operational Def'n" "<u>Intensional</u> Def'n"
- We can define L by a logical property of strings. <u>Extensional Def'n</u>

anguage or ght ulb A

$L_A = \{x \in \{1,2,3\}^*$: Both switch 1 and switch 2 are the same: both up or both down. $\}$ Q Is $\epsilon \in L$? Should it be? <u>yes</u>.

$\stackrel{?}{=} \{x \in \{1,2,3\}^*$: Either Sw 1 & Sw 2 were flicked an odd number of times or both flicked even $\}$

$$L_A \stackrel{?}{=} \{x \in \{1,2,3\}^*: \#1(x) \text{ is odd} \wedge \#2(x) \text{ is odd } \underline{\text{OR}} \;\; \#1(x) \text{ is even and } \#2(x) \text{ is even}\}$$

<u>Simpler</u> $L_A = \{x \in \{1,2,3\}^* : \#1(x) + \#2(x) \text{ is even}\}$

Similarly $L_B = \{x \in \{1,2,3\}^* : \#2(x) + \#3(x) \text{ is even}\}$

our original target $L = \{$both lights off$\} = L_A \cap L_B$.

- $L = \{x \in \{1,2,3\}^* : \#1(x) + \#2(x)$ is even <u>AND</u> $\#2(x) + \#3(x)$ is even$\}$

"Extensional Definition"

Is this alternative defn equivalent?

$L' = \{x \in \{1,2,3\}^*$ again: Either $\#1(x), \#2(x), \#3(x)$ are all odd or they are all even.$\}$

<u>Observe</u>: L' is <u>sound</u> which means everything it allows is valid.

I.e. $\boxed{L' \subseteq L}$

Why?: • If $\#1(x)$, $\#2(x)$, and $\#3(x)$ are all odd, then $\#1(x)+\#2(x)$ is even and so is $\#2(x)+\#3(x)$ even $\} \therefore x \in L$

• And if all three are even, so are both sums.

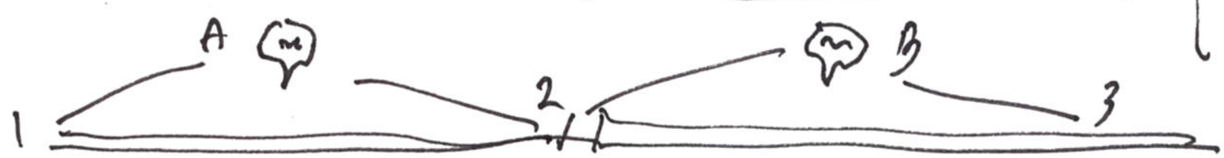
And it <u>is</u> <u>comprehensive</u> meaning $L' \supseteq L$. $\therefore$ So $\boxed{L' = L}$

1 3 2 2 w = 11332 w = 113322

w = ε has all three counts = 0. Zero <u>is</u> an even number!

- <u>Machine Definition</u> builds on the logical definition by taking into account how it affects <u>states of the system</u>.

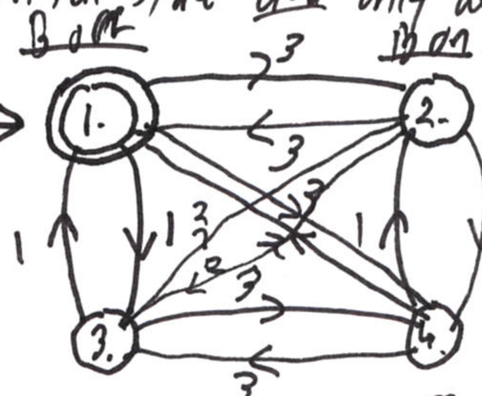
[when only finitely many states are needed, you can build a DFA]



States: {1. A off B off
{00 2. A off B <u>on</u>
01 = 3. A on B off
10
11} 4} A on B on.

(initial state <u>and</u> only desired final state)

M =



A off

$\therefore \underline{L(M)} = L$

A on

$Q = \{1,2,3,4\}$ $\Sigma = \{1,2,3\}$ $s = 1.$ $F = \{1.\}$

$\delta(1.,2) = 4.$ $\delta(4.,2) = 1.$

$\delta(2.,2) = 3.$ $\delta(3.,2) = 2.$ etc.

$\delta : Q \times \Sigma \to Q$

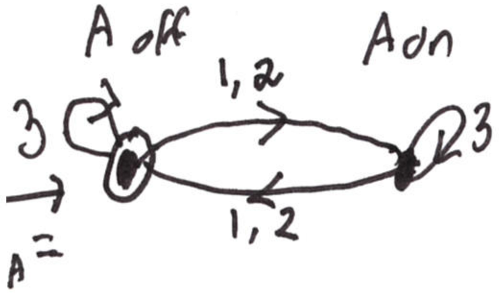
$\boxed{\delta(p,c) = q \quad (p,c,q) \in \delta}$

Alternative: $\delta = \{(1.,2,4.),(4.,2,1),(2.,2,3),(3.,2,2.)$ etc...$\}$ all in $\underline{Q \times \Sigma \times Q}$.

- A Regular Expression Definition (dare we try?!)

Recall $L = L_A \cap L_B$ where $L_A = \{ x \in \{1,2,3\}^* : x \text{ leaves light A off} \}$
i.e. $\#1(x) + \#2(x)$ is even, 3 don't care.

A DFA for L_A only:



$L_B = \{ x \in \{1,2,3\}^* : x \text{ leaves light B off} \}$
i.e. $\#2(x) + \#3(x)$ is even, 1 = don't care.

A regexp for "#(1s or 2s in x) is even":

If x has just 1s & 2s, $= \left((1 \cup 2)(1 \cup 2) \right)^*$

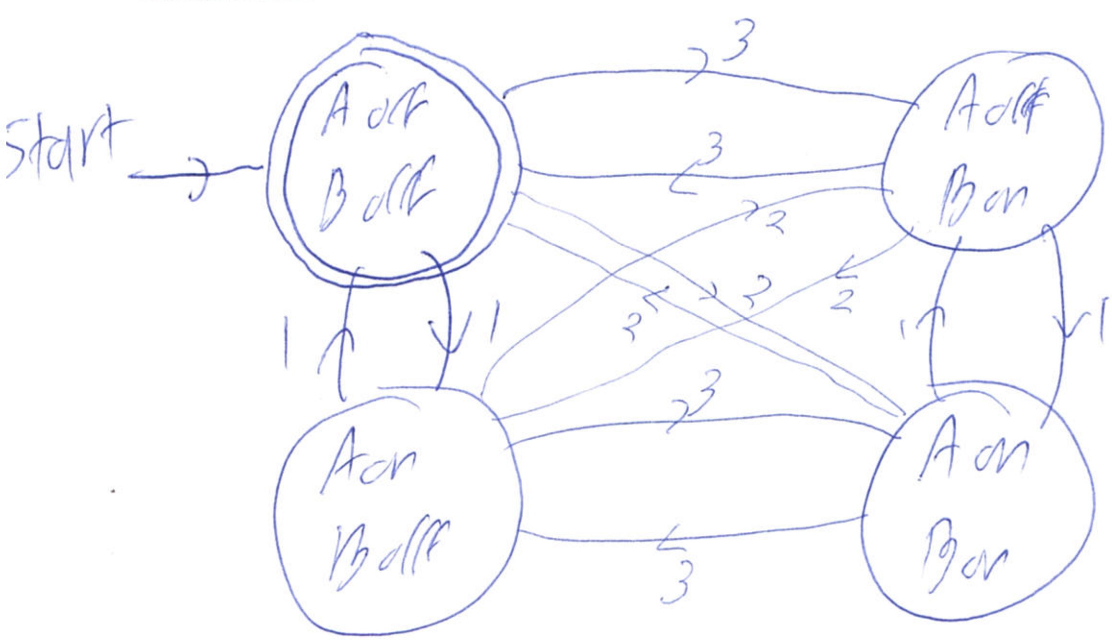
Multiplied out, $= (11 \cup 12 \cup 21 \cup 22)^*$

Include 3^* as "don't care": $r_A = \underline{\left(3^* \cdot (1 \cup 2) \cdot 3^* \cdot (1 \cup 2) \cdot 3^* \right)^*}$

Similarly, $r_B = \left(1^* \cdot (2 \cup 3) \cdot 1^* \cdot (2 \cup 3) \cdot 1^* \right)^*$ is a regexp for L_B.

Our final regexp could be $r = r_A \cap r_B$ <u>except</u>

$\boxed{\cap \text{ is not allowed as a basic Regular Operation.}}$



Extra: Larger Diagram of DFA M.

Recitations Next Week.

(1) A relation $R \subseteq A \times B$ (need not be a function $f: A \to B$ but)

always induces a function $F_R : A \to \overbrace{\mathcal{P}(B)}^{\text{Powerset}}$

[f]or all $a \in A$, $F_R(a) = \{b \in B : aRb\}$. (could be $\emptyset$) (Set of all "friends" of a person $\underline{a}$.)

Text NFA: $\delta : Q \times \Sigma \to \mathcal{P}(Q)$

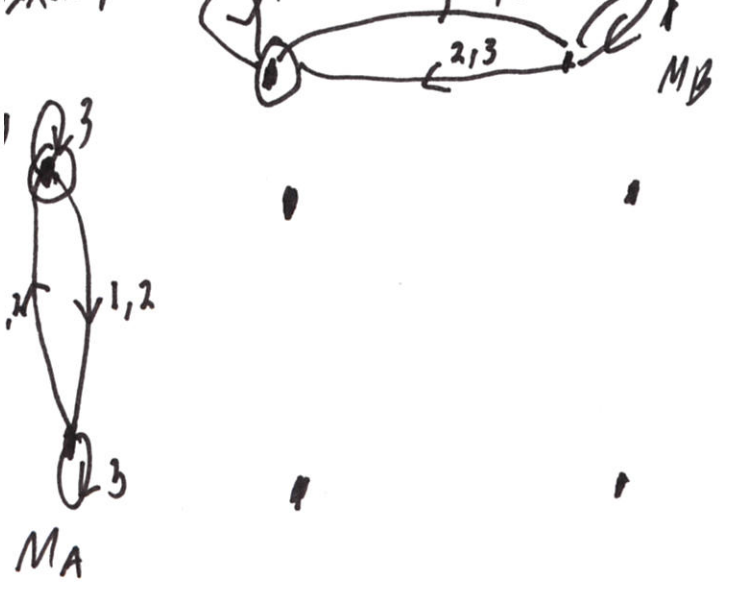
$\Updownarrow$

Lecture will give $\delta \subseteq \underbrace{Q \times \Sigma}_{\text{"A"}} \times \underbrace{Q}_{\text{"B"}}$ for both NFAs and DFAs.

(2) Cover Cartesian Product Construction (for intersection) p45 - 46 footnote)

Cover the footnote p46

[E]xample



Show geometrically how the rule

$\underbrace{\delta((p_A, p_B), c) = (\delta_A(p_A, c), \delta_B(p_B, c))}$

gives you the DFA M from the Thu Feb 4 lecture.