CSE396 Lecture Thu 2/? Spr 2018 (1)

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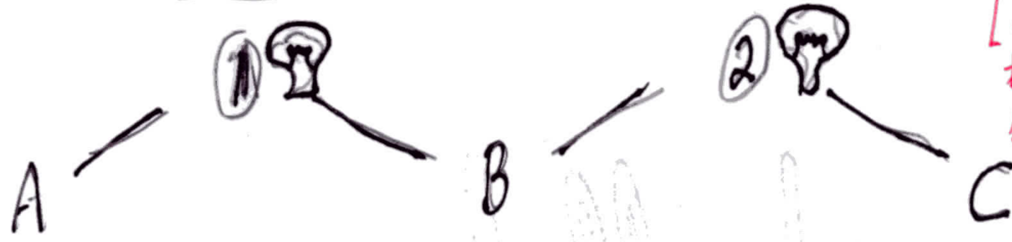
Switch



[Note: $\#A,B(x)$ equals $\#A(x) + \#B(x)$. But later $\#AB(x)$ (no comma) will count the substring AB.]

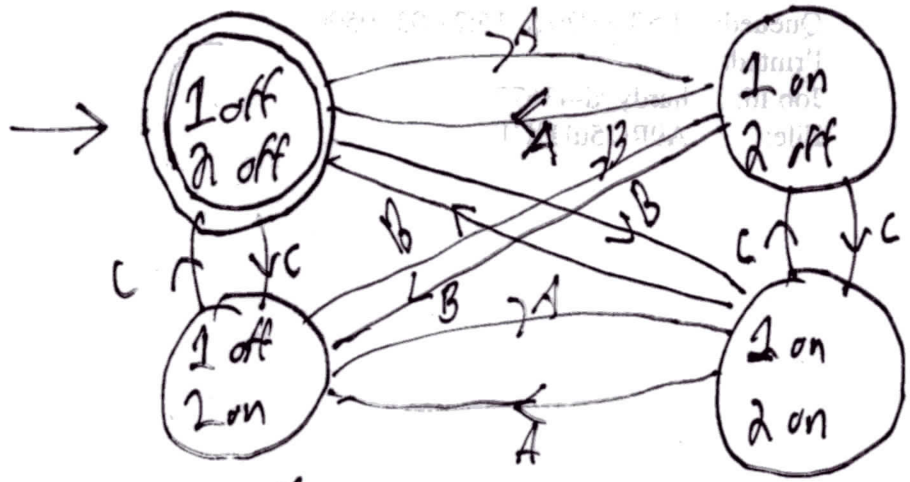
$\Sigma = \{A, B, C\}$ standing for flicks of the switches.

$x = ABAC$ leaves bulb 1 on and 2 off. Define

$L = \{x \in \Sigma^* :$ x leaves both bulbs off if they were initially off$\}$

$= \{x \in \Sigma^* : \#A,B(x)$ is even and $\#B,C(x)$ is even$\}$.

Design a DFA M s.t. $L(M) = L$. Idea: States of M ≅ States of the System

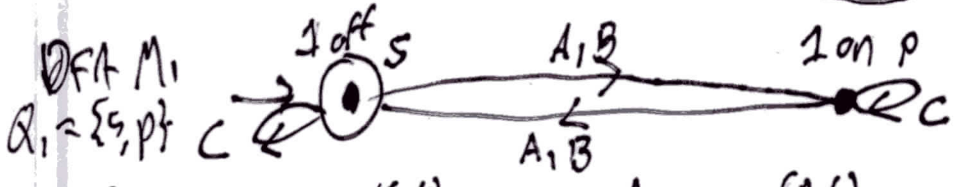


Observe: $L = L_1 \cap L_2$ where

$L_1 = \{x : x$ leaves Bulb 1 off$\}$ and

$L_2 = \{x : x$ leaves 2 off$\}$.

Hence we could build M as the Cartesian Product (for $\cap$) of DFAs M_1, M_2 enforcing L_1, L_2.



The new DFA M_3 has components:

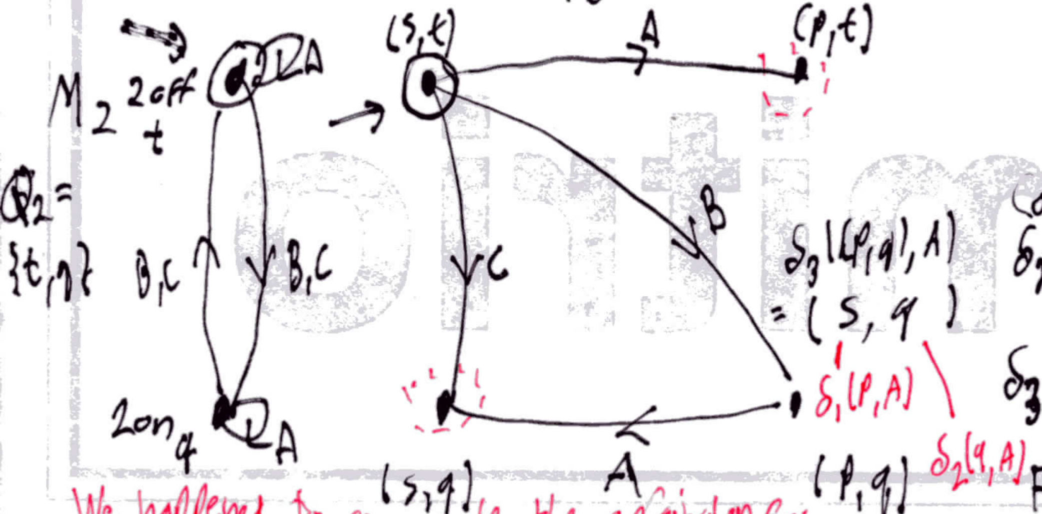
$Q_3 = Q_1 \times Q_2$, Σ stays same

$s_3 = (s_1, s_2)$ in general, (s,t) here.

Combine $\delta_1 : Q_1 \times \Sigma \to Q_1$ and $\delta_2 : Q_2 \times \Sigma \to Q_2$ to $\delta_3 : Q_3 \times \Sigma \to Q_3$ by

$\delta_3((r_1, r_2), c) = (\delta_1(r_1, c), \delta_2(r_2, c))$

$F_3 = \{(q_1, q_2) : q_1 \in F_1$ AND $q_2 \in F_2\}$



We happened to re-create the original DFA for L.

Now suppose we consider x acceptable if it leaves at least <u>one</u> light off: (2)

$\boxed{L_3' = L_1 \cup L_2.}$ In red. I write $\sim A$ for the complement of the set A.

$F_3' = \{(q_1, q_2) : q_1 \in F_1 \text{ OR } q_2 \in F_2\}$ (where $q_1 \in Q_1$ and $q_2 \in Q_2$).

Suppose $L_3'' = L_1 \setminus L_2 = \{x : x \in L_1 \text{ and } x \notin L_2\}$ presume $q_2 \in Q_2$

$\setminus$ is "\setminus" in TeX $= L_1 \cap (\sim L_2)$ $F_3'' = \{(q_1, q_2) : q_1 \in F_1 \text{ and } q_2 \notin F_2\}$

Q_4 still $= Q_1 \times Q_2$

And $L_4 = (L_1 \setminus L_2) \cup (L_2 \setminus L_1)$ $F_4 = \{(q_1, q_2) \in Q_4 : q_1 \in F_1 \text{ XOR } q_2 \in F_2\}$

$= \{x : x \in L_1 \text{ XOR } x \in L_2\}$

We write $L_4 = L_1 \Delta L_2$ and call it the <u>symmetric</u> <u>difference</u>

Δ = \Delta or \bigtriangleup

☆ We can combine any two DFAs M_1 and M_2 (that use the same Σ) into a DFA for any Boolean combination of $L(M_1)$ and $L(M_2)$. ☆

In particular, we can <u>complement</u> $L(M_1)$ by using $\widetilde{L(M_1)} = L(M_1) \Delta \underline{\Sigma^*}$ or more simply by defining $F_1' = \{q \in Q_1 : q \notin F_1\}$. Mall

I.e., interchange accepting & rejecting states.

— Extra Example —

Picture a string as a simple linear "dungeon" in which each cell may hold a sword (\$), be occupied by a dragon (d) or be empty (0). So $\Sigma = \{\$, d, 0\}$. Suppose we play by these rules:

- If a room holds a \$ and your hands are empty, you can pick it up. Cannot hold two \$ at a time.
- If a room holds d and you are holding a \$, you can use the \$ to kill the d — but since dragon hides are thick, you can't pull your \$ back out, so you leave emptyhanded.
- If a room holds d and you are emptyhanded, you're <u>Dead</u>.
- An empty room does nothing: go on to next room. • Dead guys stay dead.

$L = \{x \in \Sigma^* : x \text{ is a "survivable dungeon"}\}$

$M =$