CSE 396 — Lecture Thu Mar. 3 — Spring 2016

One More Closure Property of REG: Define

$L^R = \{x^R : x \in L\}$. $x^R = x$ written backwards

$\epsilon^R = \epsilon$ $\quad 1011^R = 1101$

For each $K \geq 1$ define $\qquad$ x is a palindrome $\iff x = x^R$.

$L_K = \{x :$ the Kth bit from the left is a 1$\}$. $\quad \alpha_K = \overbrace{(0+1)}^{K-1} 1 \overbrace{(0+1)}^{*}$

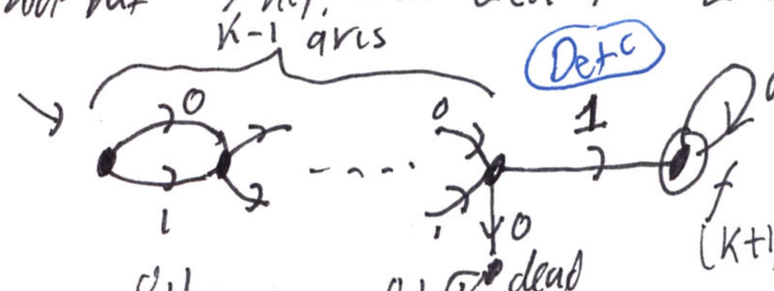
$L_K^R = \{x \in \{0,1\}^* :$ the Kth bit from the right is a 1$\}$ $\quad \beta_K \quad \overbrace{(0+1)}^{*} 1 \overbrace{(0+1)}^{K-1}$

(was "L_K" in prev lecture)

Theorem: If L is regular, then so is L^R. $\boxed{\text{i.e. REG is closed under the reversal op.}}$

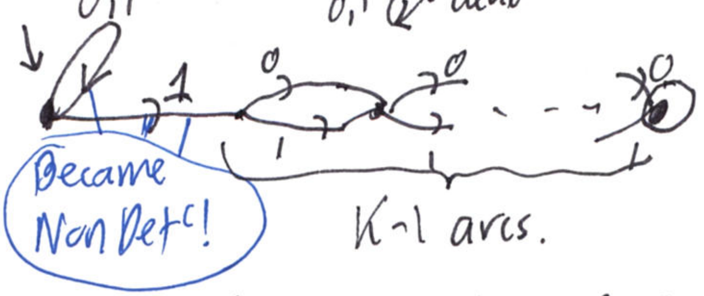
"Quick Proof": Take a regular expression α such that $L = L(\alpha)$. Write Kleene * above as shown. Define α^R = mirror image of α.
(Really needs more proof but → "hey".) Then $L(\alpha^R) = L^R$. ☒

DFA M_K
s.t. $L(M_K) = L_K$



$K+2$ states
$2K+4$ arcs
($K+1$, $2K+2$ not counting dead)

Mirror-Image NFA:
original f = new s
$L(N_K) = L_K^R$



original s = new f.

And [from addendum before] L_K^R has a PD set of size 2^K, namely $\{0,1\}^K$
∴ L^R might need exponentially more DFA "state complexity" than L!
But for regexps α and NFAs, it stays about the same.

Chapter 2 Context-Free Grammars: Invented by Noam Chomsky 1956–1959.

Example: $S \to 0S1 \mid \varepsilon$. = a CFG G.

- ε is "grammatically legal" $\varepsilon \in L(G)$.
- If w is legal, then $x = 0w1$ is legal.

$\therefore$ More legal strings: 01, 0011, 000111, ...

$\therefore$ $L(G) = \{0^n 1^n : n \geq 0\}$ which is a non-regular language.

Example 2: Variables called $\langle \text{Noun Phrase} \rangle$ $\langle \text{Noun} \rangle$ $\langle \text{Adjective} \rangle$
$\langle NP \rangle$ $\langle N \rangle$ $\langle A \rangle$

temporary start symbol   "can be a."

G_N: $\langle NP \rangle \to \langle N \rangle \mid \langle A \rangle \langle NP \rangle$. $\langle N \rangle \to$ any noun in your dictionary

$\therefore$
- $\langle N \rangle$ is legal (any noun by itself) $\langle A \rangle \to$ any adjective...
- If np is a legal noun phrase, then $\langle A \rangle$ np will be legal too, for any adjective in place of $\langle A \rangle$

As a prototype: If $\langle NP \rangle$ is legal, then so is $\langle A \rangle \langle NP \rangle$.

Aside: G^R has mirror-image productions rules: $\langle NP \rangle \to \langle N \rangle \mid \langle NP \rangle \langle A \rangle$ Like French

$\langle NP \rangle$, $\langle NP \rangle \langle A \rangle$, $\langle NP \rangle \langle A \rangle \langle A \rangle$ etc. expanding on the right.

G_V verb phrase: $\langle VP \rangle \to \langle V \rangle$ (verb) $\mid \langle VP \rangle \langle Adv \rangle$ (adverb.) $\mid \langle VP \rangle$ and $\langle Adv \rangle$

Combine: $S \to \langle NP \rangle \langle VP \rangle$.   which ties into the rules above.

$G = G_N \cup G_V \cup \{S \to \langle NP \rangle \langle VP \rangle\}$. An example derivation.

(repeated for the screen) $\langle NP\rangle \to \langle N\rangle \mid \langle A\rangle\langle NP\rangle$

$\langle N\rangle \to$ any dictionary noun, like ideas

$$S \Rightarrow \langle NP\rangle\langle VP\rangle.$$
$$\Rightarrow \langle A\rangle\langle NP\rangle\langle VP\rangle$$
$$\Rightarrow \langle A\rangle\langle A\rangle\langle NP\rangle\langle VP\rangle$$
$$\Rightarrow \langle A\rangle\langle A\rangle\langle N\rangle\langle VP\rangle$$
$$\Rightarrow \langle A\rangle\langle A\rangle\langle N\rangle\langle V\rangle\langle Adv\rangle$$
$$\Rightarrow^5 \text{Colorless green ideas sleep furiously.}$$

$\langle A\rangle \to$ any adjective

$\langle V\rangle \to$ any verb

$\langle Adv\rangle \to$ any adverb

$\Sigma =$ typewriter alphabet

A more Indicative Example: $\Sigma_G = \{0, 1, \emptyset, \varepsilon, \cup, \cdot, {}^*, (,)\}$.

$$G: E \to \emptyset \mid \varepsilon \mid 0 \mid 1 \mid (E \cup E) \mid (E \cdot E) \mid (E^*)$$

$L(G) = \{$ fully-parenthesized legal regular expressions over $\Sigma = \{0,1\}\}$.

Def^n A CFG is a 4-tuple $G = (V, \Sigma, R, S)$ where

V is a set of variables,

Σ is the terminal alphabet

$S \in V$ is the start symbol and

R is a finite set of ~~rules~~ productions of the form $V \to (V \cup \Sigma)^*$.

Next Thursday: Definition of Derivations and Ambiguity