Let \( L = \{ a^i b^i : i \geq 0 \} \). Suppose we try to prove \( L \) is not regular. Let \( s \) be in \( L \).

Take \( S = a^n \) clearly infinite. Let any \( x, y \in S \), \( x \neq y \) be given. Then we can write \( x = a^m, y = a^n \) where \( n = m + 1 \). Take \( z = b b \). Then \( x z = a^m b^2 \) and \( y z = a^n b^2 \). Then \( m + 2 \) is even \( \Rightarrow n + 2 \) is odd since \( n = m + 1 \), so \( L(xz) \neq L(yz) \). So \( S \) is PD for \( L \), \( \ldots \) etc.

ERROR: \((xz)\) is not a general choice or \( y \) did not come from \( S \).

Try \#2: Take \( S = (aa)^* \). Also clearly infinite.

Let any \( x, y \in S \), \( x \neq y \), be given. Then \( x = a^m, y = a^n \) where \( m \) and \( n \) are both even and \( n \neq m \). Take \( z = ____ 22 \). Not possible to get

In fact, all strings in \( S \) are equivalent w.r.t. to \( L \). We did get some information from the first try: \( a^{m+1} \neq a^m \) for all \( m \).

Let's put \( \varepsilon \) and \( a \) into our \( S \) representing the "\( m \) even" and "\( m \) odd" cases. Any more? Consider \( x^i = b \). Now \( b \in L \) and \( e \in L \) so \( b \neq e \), but is \( b \neq a \)? \( L = \{ a^i b^i : i \geq 0 \} \). \( \varepsilon \) is even 3, so \( ba \neq L \) while \( ad \in L \). So \( b \neq a \) via \( z = a \).

Now \( S = \{ \varepsilon, a, b, ba, ab \} \).

Is that all? Ah, \( ba \) must go to a dead state distinct from the others since none of \( \varepsilon, a, b \) is dead w.r.t. \( L \).

Since we did feel that \( L \) is regular, we can use \( S \) to help design a DFA. All strings in \( S \) must go to different states. Now we have 4 states. Do we need any more? Is \( aa \sim \varepsilon \)? Yes, so \( S(aa, a) = 5 \) is fine.

What about \( ab \)? It is in \( L \). But \( ab \neq \varepsilon \) by \( z = a \). And \( ab \) is distinctive from all the other elements of \( S \) which are in \( L \). Is \( ba \neq \varepsilon \)?
Every Statement of MNT: A language L is regular if and only if all PD sets for L are finite. When so, then there is a maximum size m of a PD set, and m is the minimum size of a DFA M s.t. L(M) = L, and that M is unique of that size. Proof not given.

Def.: A state q is equivalent to itself. (in a DFA M)

* Two states p and q are equivalent if they are both accepting or both rejecting, and for all c ∈ Σ, the states p' = S(p, c) and q' = S(q, c) are equivalent. (That is the basis for an algorithm for minimizing DFAs, but it is not on our syllabus.)

More MNT Examples: $L_{bd} = \{ x \in \{ 0, 1 \}^* : x \text{ is a survivable dungeon when you may hold any # of spears}, \}$

Prove that $L_{bd}$ is not regular: Take $S = \{ * \}$. Clearly S is infinite. Let any $x \in S$, $x \neq y$ be given. Then there are $m, n \geq 0$ such that $x = y^m$, $y = y^n$, and wlog $m < n$. Take $z = d^n$. Then $z^2 = y^{2n} \notin L_{bd}$ because $m < n$, so the player gets killed by the $n+1$st dragon. But $yz = y^{2n} \in L_{bd}$.

Thus $L_{bd}$ is not regular. Take $x \neq y$ be given. Because $x \neq y$, there is a bit place i s.t. $x_i \neq y_i$. Then $x^i \in L_{bd}$ and $y^i \notin L_{bd}$, so that many states are needed. Thus, $S = \{ 0, 1 \}^*$ is a PD set for $L_{bd}$ of size $S$, so that many states are needed. Notice from 0.

Claim: $S = \{ 0, 1 \}^*$ is a PD set for $L_{bd}$ of size $S$, so that many states are needed. Let any $x \in S$, $x \neq y$ be given. Because $x \neq y$, there is a bit place i s.t. $x_i \neq y_i$. Take $z = 0^i$. Suppose "x" is the string such that $x_i = 1$, $y_i = 0$. Then $y^i$ has a 1 in i

Abstract Example: Define $L_K = \{ x \in \{ 0, 1 \}^* : |x| \leq K \}$ and the K-th bit from right is $0$.

NFA for $L_K$ has $K+1$ states. How many states does a DFA M need? $N = 2^K$ states

Claim: $S = \{ 0, 1 \}^*$ is a PD set for $L_K$, of size $S^K$, so that many states are needed. Let any $x \in S$, $x \neq y$ be given. Because $x \neq y$, there is a bit place i s.t. $x_i \neq y_i$. Take $z = 0^i$. Suppose "x" is the string such that $x_i = 1$, $y_i = 0$. Then $y^i$ has a 1 in i

Please let me know if there's anything else I can assist you with!