CSE 396 Lecture Tue Mar. 6 Spring 2018

Top Hat # 8479

Extra/FYI/Solution to Ch1 problems mid-40s.
Consider $\Sigma = \{0,1\}$ and any language $L \subseteq \Sigma^*$

For all x in Σ^* define: $row_L(x)$ to be the infinite vector

(all $z \in \Sigma^*$)

$row_L(x) = [L(x), L(x0), L(x1), L(x00), L(x01), \dots, L(xz), \dots]$

Then $x \equiv_L y$ if and only if $row_L(x) = row_L(y)$. A PD set for L has all distinct rows.

Full Myhill-Nerode Theorem: L is regular $\Leftrightarrow$ there are only finitely many distinct $row_L(x)$ vectors $\Leftrightarrow$ all PD sets for L are finite. We have seen the $\Rightarrow$ direction in contrapositive.

Proof ($\Leftarrow$): Suppose there only finitely many distinct rows.
Define $M = (Q, \Sigma, \delta, s, F)$ as follows: $Q = \{\text{distinct rows}\}$,

$s = row_L(\epsilon)$. both finite!

$$\boxed{\delta(row_L(x), c) = row_L(xc).}$$

$F = \{row_L(x) : x \in L\}$

since $row_L(x)$ itself is the object, it doesn't matter "which x".

Then $L(M) = L$, and since Q is finite, M is a DFA, so $L \in REG$.

Corollary: The number K of distinct rows, which equals the max size of a PD for L, equals the min # of states needed by a DFA for L, and the M in the proof is the unique DFA achieving it.

More Non(?) regular Language Examples: Consider

$$L_1 = \{ \overset{u}{x} \cdot \overset{v}{y} : \#0(\overset{u}{x}) = \#1(\overset{v}{y}) \}$$

$$L_2 = \{ x0y : \#0(x) = \#1(y) \}.$$

Intension (intent) vs Extension (semantics) actual behavior.

"Poof" that L_1 is non-regular: Take $S = 0^*$. clearly infinite. Let any $x, y \in S$, $x \neq y$, be given. Then we can write $x = 0^m$, $y = 0^n$, where w.l.o.g. $m < n$. Take $z = 1^n$. Then $xz = 0^m 1^n \notin L_1$ since $n \neq m$ (Wrong - why?) but $yz = 0^n 1^n \in L_1$. So $L(xz) \neq L(yz)$, and since S infinite, this proves the "theorem".

Let's try $m=3, n=5$ $x = 000$ $z = 11111$. Is $xz = 00011111 \in L_1$?
$w = 00011 \cdot 111$

Re-define $L_1 = \{w : w$ can be broken as $w =: u \cdot v$ s.t. $\#0(u) = \#1(v)\}$.
Thus the "poof" is wrong. In fact, L_1 is regular!

$\boxed{L_1 = \Sigma^*}$: Given any $w \in \Sigma^*$, $|w| = n$.
For each i, $0 \leq i \leq n$, define $f(i) = \#0(x_1 \cdots x_i) - \#1(x_{i+1} \cdots x_n)$

$f(0) = -\#1(x)$ which is ≤ 0 $f(i)$ steps up by $+1$ (or 0 or -1) from $f(i-1)$
$f(n) = \#0(x)$ which is ≥ 0. $\therefore$ there is an i s.t. $f(i) = 0$. Take $u = x_1 \cdots x_i$, $v =$ rest.
So $w \in L$, and since $w \in \Sigma^*$ is arbitrary, $L = \Sigma^*$ which is regular: $(0 \cup 1)^*$.

This proof does not work for L_2 because the "step up to 0" could be when $x_i = 1$. Then we don't get the needed 0 in $x0y$.

Re-define $L_2 = \{ w \in \{0,1\}^* :$ w can be broken as

$$w =: [redacted] \cdot 0 \cdot v \quad \text{st.} \quad \#0(u) = \#1(v) \}.$$

<u>Proof that L_2 is non-regular</u>: Take $S = 0^*$. (clearly infinite.)

Let any $x, y \in S$, $x \neq y$, be given. Then we can write $x = 0^m$, $y = 0^n$ where <u>wlog</u> $m < n$. <u>Take $z = 1^{n-1}$</u>

Then $xz = 0^m 1^{n-1} \notin L_2$ since <u>any possible parse $xz =: u0v$</u> (Saying $n > m$ by itself: not enough:) <u>uses one of the 0s from the 0^m part, but then $\#0(u) \le m-1$ while $\#1(v) = n-1$</u> ~~They can't be equal since m<n~~. <u>They can't ~~be equal since~~ → implies $m-1 < n-1$ since $m<n$?</u>. Whereas $yz = 0^n 1^{n-1} \in L_2$, because it parses as $0^{n-1} \cdot 0 \cdot 1^{n-1}$. So $L(xz) \neq L(yz)$. Since $x, y \in S$ are arbitrary and S is infinite, $L_2 \notin REG$ by MNT

<u>Ch 2 Preview</u>

What kind of formalism can describe languages like L_2?

Note: $\varepsilon \notin L_2$. $0 \in L_2$ acts as a base case.

$w = 10$ OK. $w = \overset{0}{0}\overset{v}{0}$ ($u = \varepsilon$) OK? <u>yes</u> since it parses as $w = \underset{u}{\varepsilon} \cdot \underset{0}{0} \cdot \underset{v}{0}$

$w = 01$? ∴ Not OK. Must <u>balance</u> by sending the base 0 to $w = \underline{0} \cdot 0 \cdot 1$.

$S \to 0$
$S \to 1S$
$S \to S0$
$S \to 0S$[redacted]

Abbreviate as

$G: S \to 0 \mid 1S \mid S0 \mid 0S1$ <u>Claim</u>: $\boxed{L(G) = L_2}$.

$G': S \to 0S0 \mid 1S1 \mid \varepsilon \mid 0 \mid 1$. $L(G') = \underline{PAL}$ the set of palindromes