CSE 396 Lecture Thu 4/4 Spring 2019

Top Hat 6033

"Adversary Argument" Script for the CFL Pumping Lemma

Example: $L = \{a^i b^j c^k : i < j \text{ \&\& } j < k\}$.

Adv: "I have a CFG G s.t. $L(G) = L$."

You: "Give me the $N = 2^{|V|}$ from a CFG G' in ChNF s.t. $L(G') = L \setminus \{\epsilon\}$"

Adv: "N"

You: "We take $x =$ ______. Note $x \in L$ and $|x| > N$. Now give us a breakdown $x =: yuvwz$ s.t. $|uvw| \leq N$ and at least one of u, w is not ϵ."

Adv: "(you must be prepared for any answer that meets the conditions)"

You: [Break into case analysis and show in each case there is an i such that $x^{(i)} = yu^ivw^iz$ is not in L, because ______

______

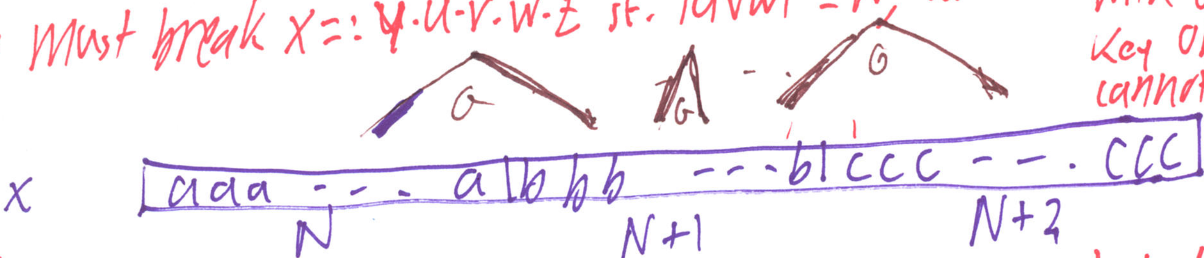
Your report concludes this all contradicts the CFL PL, so G does not exist, so L is not a CFL. ☒

Example: Adv says "N".

You take $x = a^N b^{N+1} c^{N+2}$. Then $x \in L$ and $|x| > N$.

Adv: Must break $x =: y \cdot u \cdot v \cdot w \cdot z$ st. $|uvw| \leq N$, $uw \neq \epsilon$.

Divide into cases with aid of pictures. Key Obs: compass cannot write in both the a's and c's region.



Cases:

① The compass does not write in the a's region - but does write in the b's region or c's.

② The compass does not write in the c's region.

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You state these but the details still belong to Adv. Your cases must however be exhaustive.

In case ①, choose $i = \underline{0}$: $x^{(0)} = yvz$ either • subtracts at least 1 b or • subtracts no b's but a c.

Thus either the "$i < j$" condition is violated, or the "$j < k$" one is violated. Either way, $x^{(0)} \notin L$.

In Case ②, you choose $i = \underline{2}$. Then $x^{(2)} = yuuvwwz$ either

- adds at least one **b** — Then since $x^{(2)}$ cannot add any c's by being in case ②, the $j < k$ condition is violated.

or

- adds at least one a **and** does not add any b's — Then $x^{(2)}$ adds only 1 or more a's, so the "$i < j$" is violated, so $x^{(2)} \notin L$ here too.

So $x^{(2)} \notin L$ in case ②, so in both major cases we found an i s.t. $x^{(i)} \notin L$ so L is not a CFL by the CFL PL! ☒

Again you need to do subcase analysis – though you can say that since the c's are not written in, the $N+2$ c's are a "sitting duck" for pumping-up elsewhere.

Example: A Three-Way Comparison: Which languages are CFLs?

$L_1 = \{0^a 1^b 0^a 1^b : a, b \geq 1\}$.

$L_2 = \{0^a 1^a 0^b 1^b : a, b \geq 1\}$

$L_3 = \{0^a \underbrace{1^b 0^b} 1^a : a, b \geq 1\}$.

L_1 is visually similar to strings like

$\underbrace{\{\{\cdots\{}_{a} \underbrace{((\cdots(}_{b} \underbrace{\}\}\cdots\}}_{a} \underbrace{))\cdots)}_{b}$

Unboundedly-many crossing dependencies.

L_3 is like $\underbrace{\{\{\{\cdots}_{a} \underbrace{(((}_{b}\cdots\underbrace{)))}_{b} \underbrace{\}\}\}}_{a}$

L_2 is like two properly nested program routines side by side. And L_2 is also a CFL (like on HW5).

And L_1 is not a CFL. Intuition:

$x = \overbrace{\boxed{00\cdots0}\,\boxed{11\cdots1}}^{G}\,\overbrace{\boxed{00\cdots0}\,\boxed{1\cdots11}}^{G}$

Compass cannot touch both 0s intervals, nor both 1s intervals, but must write somewhere.

L_3 does obey proper nesting and it is a CFL: $S \rightarrow 0S1 \mid 0T1$, $T \rightarrow 1T0 \mid \underline{10}$.

or put the full balanced parentheses grammar here.

An aspect of $\{a^n b^n c^n\}$ not being a CFL: A programming language grammar cannot enforce that the number of parameters in a (C++) method is the same

(a) where it is declared in .h

(b) where it is defined .cpp

and (c) where it is called in user code. Multiple calls would be "d^n", "e^n", etc.

Is there a machine model that can check $\{a^n b^n c^n d^n e^n\}$ etc? Yes. A Turing Machine

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