"Adversary Argument" Script for the CFL Pumping Lemma

Example: \( L = \{ a^i b^j c^k \mid i < j \text{ and } j < k \} \)

Adv: "I have a CFG \( G \) s.t. \( L(G) = L \)."

You: "Give me the \( N = 2^{11} \) from a CFG \( G' \) in ChNF so \( L(G') = L \)."

Adv: "\( N \)"

You: "We take \( x = \ldots \). Note \( x \in L \) and \(|x| > N \). Now give us a breakdown \( x = yuvwz \) st. \(|uvw| \leq N \) and at least one of \( u, v \) is not \( \varepsilon \)."

Adv: "(you must be prepared for any answer that weakens the condition)"

You: [Break into case analysis and show in each case there is an \( i \) such that \( x^{(i)} = yuvwz \) is not in \( L \), because]

Your report concludes this all contradicts the CFL PL, so \( G \) does not exist, so \( L \) is not a CFL.

Example: Adv says "\( N \)"

You take \( x = a^N b^{N+1} c \). Then \( x \in L \) and \(|x| > N \).

Adv: "Must break \( x = yuvwz \) st. \(|uvw| \leq N \), \( uw \neq \varepsilon \). Divide into cases with aid of pictures."

Key Obs: Compass cannot write in both \( a \) and \( c \) region.

Cases:
1. The compass does not write in the \( a \) region.
2. The compass does not write in the \( c \) region.

In case 2, choose \( j = 0 \), \( x^{(i)} = yvz \) either subtracts at least \( ab \) or subtracts no \( b \) but \( c \).

Thus either the "\( i < j \)" condition is violated, or the "\( j < k \)" condition is violated. Either way, \( x^{(i)} \notin L \)."
In Case 2, you choose $i = 2$. Then $x^{(2)} = yuvvwz$ either
- adds at least one $b$
- adds at least one $a$
  and does not add any $c$.

Then $x^{(2)}$ adds only 1 or more $a$s, so $x^{(2)} \notin L$ here too, both major cases we found an i st.: $x^{(2)} \notin L$, so $L$ is not a CFL by the CFL PL.

Example: A Three-Way Comparison: Which languages are CFLs?

$L_1 = \{0^n 1^n 0^n 1^n : n \geq 1\}$
$L_2 = \{0^n 1^n 0^n 1^n : n \geq 1\}$
$L_3 = \{0^n 1^n 0^n 1^n : n \geq 1\}$

$L_1$ is visually similar to strings like

Unboundedly - many crossing dependencies.

$L_2$ is like two properties nested program-reuten side-by-side and $L_2$ is also a CFL (like an HVS).

$L_3$ does not have proper nesting and it is a CFL: $S \rightarrow OS110T1$

An aspect of $\{a^n b^n c^n\}$ not being a CFL: A programming language grammar cannot enforce that the number of parameters in a (C++) method is the same where it is declared in .h
- where it is defined - .cpp
- where it is called in user code. Multiple calls would be "m, n", etc.

Compass cannot paint both Os intervals nor both Is intervals, but must write somewhere.

Again you need to do sub-case analysis - though you can say that since the $c$'s are not written in the $N+2$ c's are a "fishing duck" for pumping-up elsewhere.