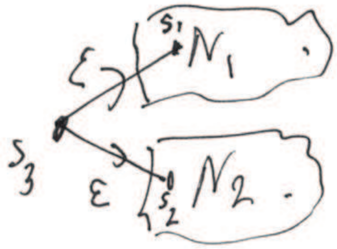
HW Questions:

$S \to [Adj]\, S \mid \underline{noun}_1 \mid noun_2 \mid noun_3)$ — Is OK for short form if noun is considered a shortcut for terminals

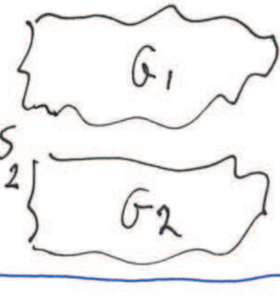
~~$S \to \underset{\sim}{\varepsilon} \mid \$ \to \underset{\sim}{\emptyset} \mid \underset{\sim}{Q} \mid \underset{\sim}{1}$~~

$S_0 \to \underset{\sim}{\emptyset}$ $\quad S_2 \to \underset{\sim}{Q}$

$S_1 \to \underset{\sim}{\varepsilon}$ $\quad S_3 \to \underset{\sim}{1}$



$S_3 \to S_1 \mid S_2$



$G_1 = (V_1, \Sigma, R_1, \underline{S_1})$

$G_2 = (V_2, \Sigma, R_2, \underline{S_2})$

$G_3 = (V_3, \Sigma, R_3, S_3)$ where $V_3 = V_1 \cup V_2 \cup \{S_3\}$, Σ same, S_3 new start and $R_3 = R_1 \cup R_2 \cup \{S_3 \to S_1 \mid S_2\}$. This level of formality is a "useful extra."

G_3 is still in short form because
- No old rules were changed or augmented.
- The new rule $S_3 \to S_1 \mid S_2$ for the new var S_3 is OK.

Side Note

Trans Closure adds $A \to C$ but not the reverse.

Lecture Proper – From 11:30

A Different View of the CFL PL as an "Adversary Argument". — Not explicit in the text.

Example 2.38 a little different

$$L = \{a^i b^j c^k : \overbrace{i < j < k}^{i<j \,\wedge\, j<k}\}$$

Text: non-strict ≤

Prove that L is not a CFL. ① Use Script. or ② Imagine an Adversary claiming he has a CFG G s.t. $L = L(G)$.

I. Tell me a number $p \geq 2^k$, where k is the # of vars in your CFG (after converting it to Chomsky NF) ← not needed in text. (Text b^k)

Adv: "p"

II. You take $\underline{x}$ (Ⓢ text) $= a^p b^{p+1} c^{p+2}$. For the record, is $s \in L(G)$? Adv must say yes.

prev lecture, but follow text now.

III <u>You say</u>: Give me your breakdown $s =: uvxyz$
such that $|vxy| \le p$ and v, y not both ε.

<u>Adv</u> → Go behind closed doors $s = \boxed{a^p \mid b^{p+1} \mid c^{p+2}}$

$vxy =$ — $\le p$ — or ...

The heart of the argument

<u>Heart</u> is to show that <u>whatever breakdown</u> <u>Adv gives</u>, you can prove an earlier contradiction in the testimony for $L = L(G)$.

<u>Either</u>:
- vxy contains only b's [Between v and y there is at least one b, say r b's total]
- or vxy has an a, (but must have no c's)
- or vxy has a c, hence no a's.

These cases are mutually <u>exhaustive</u>.

<u>In the first case</u>, "pump down" to $s^{(0)} = uxz$ $\quad s^{(i)} = uv^i x y^i z$

Since vxy had no a's or c's, only some number $r > 0$ of b's,

$$uxz = a^p b^{p+1-r} c^{p+2} \notin L \text{ since } p+1-r \le p, \text{ not } > p.$$

Thus the CFL PL is contradicted in this case.

<u>In the second case</u>, vxy has no c's. Let v, y collectively have $\left\{ \begin{array}{l} r \text{ } a\text{'s} \\ q \text{ } b\text{'s} \end{array} \right.$

Note: Either r, q could be 0, either v, y could but ε, but not <u>all</u>

Then for $i \ge 2$, $s^{(i)} = uv^i x y^i z$ has: $p + (i-1)r$ a's
$p+1 + (i-1)s$ b's
still $p+2$ c's.

Since at least one of r, q is ≥ 1, when <u>$i = 3$</u>, $s^{(3)}$ has at least $p+2$ a's and/or $p+2$ b's.

Thus you can't maintain both $i < k$ and $j < k$ in "$a^i b^j c^k$" form

So $s^{(3)} \notin L$. [See note at end] — [with or without reducing b's]

<u>In the third case</u>, <u>pumping down</u> reduces the # of c's so $s^{(0)} \notin L$ again. So all 3 cases violate $(\forall i)\ s^{(i)} \in L$.

∴ Adv cannot possibly have G. ∴ G does not exist, ∴ L is not a CFL. ☒

Text's other main example is $L = \{ww : w \in \{a,b\}^*\}$ over $\Sigma = \{a,b\}$. ③

Proof boils down to showing

$L' = \{a^m b^n a^m b^n\}$ is not a CFL. [whereas   not nested.

$L'' = \{a^m b^n a^n b^m\}$ <u>is</u> a CFL!]   nested

For L', (also L) ~~ask~~ p, take $s = a^p\, b^p\, a^p\, b^p$.

① ② ③ ④

Quick idea: a $\wedge$ of width $\leq p$ must touch at least one of intervals ①, ②, ③, or ④, but cannot keep its ~~odd~~ even partner in balance.

$\therefore s^{(0)} \notin L'$

$\therefore L'$ is not a CFL.

L'' has the two-tier CFG $S \to \underbrace{aSb}_{\text{outside}} \mid T,\ T \to \underbrace{bTa}_{\text{inside}} \mid \epsilon$.

Regardless, L'' isn't regular either.

Q What kind of liberalization of a DFA $M = (Q, \Sigma, \delta, s, F)$ can recognize these languages?

① Allow M to <u>change</u> a char

② Allow M to move <u>left</u>

Allowing ① or ② alone doesn't help.

Allowing <u>both</u> defines a Turing Machine → Ch 3.

$\boxed{\text{End Note}}$ In the third case at the end of the proof, reducing c's <u>could</u> reduce b's as well — picture "$\wedge_{vxy}$" as shown (was "..." on p2). This preserves the "$j < k$" inequality, but then "$i < j$" <u>fails</u> since a^p is untouched. <u>That</u> finishes it. ☒

$a^p \mid b^{p+1} \mid c^{p+2}$