CSE 396 Lecture Thu Apr 5 Spr 2018

Top Hat 8242

$\Sigma = \{a, b\}$

$$G = \begin{array}{l} S \to \varepsilon \mid b \mid AS \mid SC \\ A \to a \mid \cancel{bCaA} \\ C \to aS \mid \cancel{CC} \end{array}$$

Target: $E = \{x \in \{a,b\}^* : x$ does not have a "bb" in it$\}$.

Prove that $L(G) \subseteq E$, ie. G is sound for that target. Script:

I: Assign a target property P_A to every variable A (not just S).

P_S = "Every $x \in \Sigma^*$ that I derive does not have a 'bb' in it."

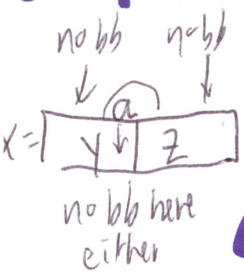
P_A = "P_S & every y I derive ends in a."

P_C = P_S & "every z I derive begins with an a."

II: For each rule of the form $B \to \vec{X}$, show that if the variables on the RHS in $\vec{X}$ obey their properties (for substrings of a terminal string x derived starting with that rule) then the property P_B on LHS is upheld by X.

$S \to \varepsilon$, $S \to b$: Immediate (base cases) (for P_A).

$S \to AS$: Suppose $S \Rightarrow^* x$ using this rule first (utrf).
Then $x = y \cdot z$ where $A \Rightarrow^* y$ and $S \Rightarrow^* z$.

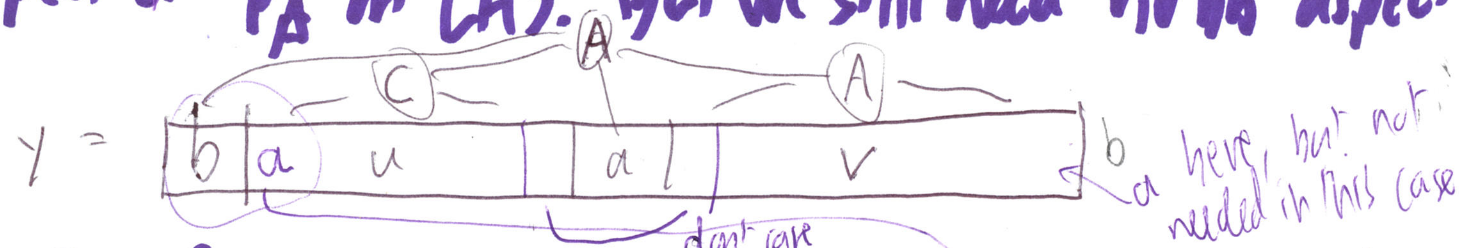


By IH P_A for y and P_S for z on RHS, y and z each have no bb, and y ends in a. Hence x cannot have a bb, which upholds P_S on LHS.

$S \to SC$: Suppose $S \Rightarrow^* x$ utrf. Then $x = yz$ where $S \Rightarrow^* y$, $C \Rightarrow^* z$. By IH P_S, P_C on RHS, z begins with 'a' and ... Symmetrical

$A \to a$: Immediate, both for not having a bb in it and for 'ends in a'. (Base case for A).

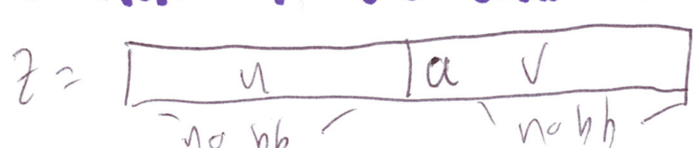
$A \to bCaA$: Suppose $A \Rightarrow^* y$ utrf. Then $y =: buav$ where $C \Rightarrow^* u$ and $A \Rightarrow^* v$. By IH P_A on RHS, v ends in a. Hence y ends in a, which upholds that aspect of P_A on LHS. But we still need "no bb" aspect.



By IH P_C on RHS, u begins with a, and u & v have no internal bb by the "P_S" part of both P_C and P_A.

$C \to aS$: Suppose $C \Rightarrow^* z$ utrf. Then $z = ax$ where $S \Rightarrow^* x$. By IH P_S on RHS, x has no bb, and we added an "a" so z has no bb either. And z begins with a. $\therefore$ P_C on LHS.

$C \to CC$: Suppose $C \Rightarrow z$ utrf. Then $z = uv$ where $C \Rightarrow^* u$ and $C \Rightarrow^* v$. By IH P_C on RHS (twice), u and v have no bb and v begins with a. Picture:



And z begins with a since v begins with a

Hence z has no bb (whew!) $\therefore$ P_C on LHS. Since we upheld P_S, P_A, and P_C in all rules, $L(G) \subseteq E$. ☒

Is G comprehensive? I.e., is $E \subseteq L(G)$?

Consider: $x = bab$ is in E. Is it derivable in G?

Can't start $S \Rightarrow \epsilon$ or $S \Rightarrow b$.

If it starts $S \Rightarrow AS$, then A must derive $bCaA$ and too many letters.

If it starts $S \Rightarrow SC \Rightarrow b\underline{C} \Rightarrow \underline{baS} \Rightarrow bab$ ✓. ($C \to aS$)

We have $\boxed{S \Rightarrow^* aS}$ and $\boxed{S \Rightarrow^* baS}$ and $S \Rightarrow b$.

$\therefore$ S derives all of $(a + ba)^*(\epsilon + b)$

which "we know" is a regular expression for E.

Hence G is comprehensive. **Proof was "ad hoc".**

Since we didn't use the rules $A \to bCaA$ and $C \to CC$ for this proof, we can delete them.

SI for failure of $E \subseteq L(G)$.

$G = S \to 01S \mid 10S \mid 1S0 \mid 0S1 \mid S01 \mid S10 \mid \epsilon$.

$E = \{x \in \{0,1\}^* : \#0(x) = \#1(x)\}$.

$x = 1100011$

P_S = "Every x I derive belongs to E."

Strengthen P_S to read:

$P'_S = P_S$ & "... either begins with 2 different letters, ends with 2 different letters, or has 2 different letters front and back."

Upheld (immediately) by all rules, so $x \notin L(G)$.