A k-tape TM $M = (Q, \Sigma, \Gamma, \delta, q_0, \delta, F)$ has:

$$\delta \subseteq (Q \times \Gamma^k \times \Gamma^k) \times \Gamma^k \times \Gamma^k \times \{L, R, S\} \times \Gamma^k \times Q.$$ 

When $k=2$, a typical instruction is $(p, c_1, \alpha, d_1, D_1, \gamma) \xrightarrow{c_2, d_2} (p', q, \alpha, d_2, D_2, \gamma)$.

The input tape is indexed as “Type 1.”

It is read-only if always $d_1 = c_1$ in every instruction.

It is one-way if $D_1$ never is $L$. ($S$ or $R$ are OK).

real-time if always $D_1 = R$. (DFTAs and NFTAs are real-time.

Def: (replacing notation in text §2.2): A 2-tape TM is a Pushdown Automat (PDA).

- its input tape is read-only and one-way
- its second tape behaves as a stack, meaning $D_2 = L \Rightarrow d_2 = \gamma$ in every instruction (“pop”)

By the text’s normal form, for every state $q$ other than $q_{acc}$, $q_{rej}$ and sequence $c_1, \ldots, c_k$ of choices, $\delta$ has at least one instruction beginning $(q, c_1, \ldots, c_k)$. If there is exactly one instruction then $M$ is deterministic. ($\delta$ is a function).

These def’s can’t thru to define deterministic pushdown automata (DPDA’s) and the default nondeterministic ones (NPDA’s). Now we can state:

Theorem: A language $L \subseteq \Sigma^*$ is context-free if and only if there is an NPDA $N$ st. $L = L(N)$. (Proof in §2.2, skipped).

Def: A language $L$ is a deterministic CFL (DCFL) if there is a PDA $M$ such that $L(M) = L$. But $\{a^ib^jc_k : i+j+k \geq 1\}$ is a CFL whose complement is not a CFL.
Defn: A TM $M$ obeys **good housekeeping** if:

- it is in the text's normal form with $q_{ac}$ and $q_{re}$ the only halting states.
- $M$ never writes an actual blank in between two nonblank characters. Instead it treats $\lambda$ as a substitute blank (like $\lambda$ on tape 2 for space in $\Gamma$).
- Then its tapes never have internal blanks. (Nor a char after one before, a blank on right on left.)
- Wherever $M$ wants to halt, and possibly leave an output $y = f(x)$ on Tape 2 (or some other designated output tape), $M$ erases all other tapes and non-output chars using actual blanks, re-winds to scan the first char of $y$ ($y = \varepsilon$, and finally goes to $q_{ac}$ or $q_{re}$).

If $M$ only wants to define a language $L(m)$, it accepts $x$ when $x \in L(m)$, it rejects $x$ when $x \notin L(m)$.

Defn: A configuration, also called instantaneous description (ID) of a TM $M$ at any point in a computation on an input $x$, specifies:

- the current state $q$ of $M$
- the current non-blank tape contents $\overrightarrow{w} = (w_1, \ldots, w_k)$ on each tape, and
- the current head positions $(h_1, \ldots, h_k)$ relative to the start of $w_j$ on each tape.

We write an ID as $I = \langle q, \overrightarrow{w}, \overrightarrow{h} \rangle$. When $k = 1$ we can code this up as a single string $\langle uqcv \rangle$ where $w = ucv$ over alphabet $Q \cup \Gamma \cup \{\varepsilon, \lambda\}$.

Defn: Given IDs $I$ and $J$, write $I \rightarrow^1 J$ (I can go to J by one step on M) if $M$ has an instruction applicable to M whose execution gives the ID $J$.

Also write $I \rightarrow^0 I$, $I \rightarrow^k K$ if there is an ID $J$ st. $I \rightarrow^1 J$ and $J \rightarrow^k K$, and $I \rightarrow^k J$ if $I \rightarrow^k J$ for some $k$. A tape TM with $\Gamma$ is $I_0(x) = \lambda$, $J = q_{ac}$.

Defn: $x \in L(m)$ if $I_0(x) \rightarrow^* J$ for some ID $J$ with $q = q_{ac}$, where $I_0(x)$ has $x$ on tape 1, $h_1$ first bit of $x$, all other tapes blank.