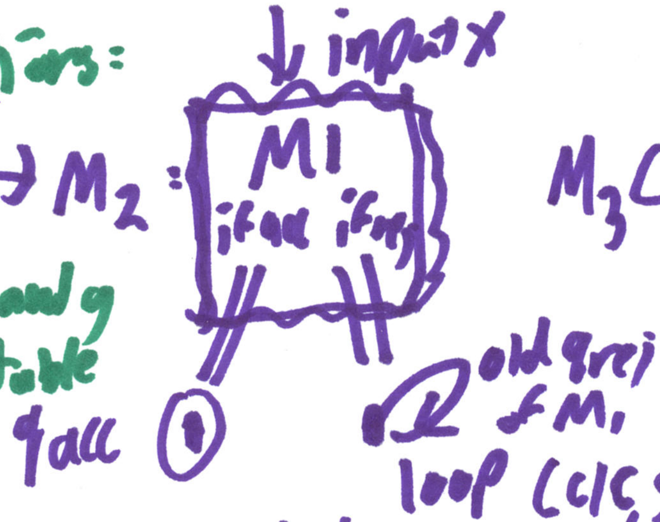
CSE 396 Lecture Tue 5/8 Spr 2018 (1)

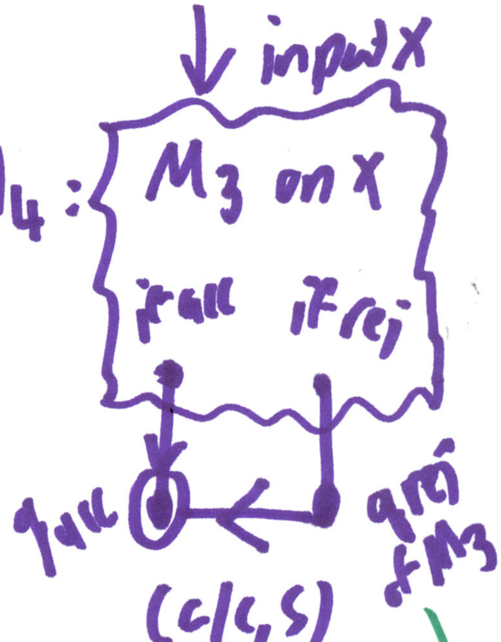
Top Hat 6626 $ALL_{TM} = \{\langle M \rangle : L(M) = \Sigma^*\}$ and $\underline{TOT} = \{\langle M \rangle : M \text{ is total}\}$ are mapping equivalent.

Constructions:

$M_1 \xrightarrow{f} M_2$:



$M_3 \xrightarrow{g} M_4$:



Clearly f and g are computable

Analysis: On any x, don't halt.

$M_1(x)$ accepts $\Rightarrow$ $M_2(x)$ halts and accepts

$M_1(x)$ rejects or doesn't halt $\Rightarrow$ $M_2(x)\uparrow$

$\therefore$ M_2 is total $\Leftrightarrow$ $L(M_1) = \Sigma^*$ $\therefore$ $ALL_{TM} \leq_m TOT$

$M_3(x)$ halts $\rightarrow$ M_4 accepts x in either of two ways

$M_3(x)\uparrow$ $\Rightarrow$ $M_4(x)$ doesn't halt either. $L(M_4) = \Sigma^* \Leftrightarrow M_3$ is total.

$\therefore$ $TOT \leq_m ALL_{TM}$

$\therefore$ $ALL_{TM} \equiv_m TOT$ Both are neither ce nor co-ce.

Intuition: Both require "$(\forall x \in \Sigma^*)$" $(\exists$ a computation $\vec{c})$ $\vec{c}$ is valid and accepts / halts.

Turing Machine Computations As Strings

Enough to do for 1-tape TMs. (nondeterministic OK)

An ID can be represented as $I = \underbrace{[}_{\text{extra}}\, u\, q\, c\, v \underbrace{]}_{\text{extra}}$

over the alphabet $Q \cup \Gamma \cup \{[,]\}$

$M = (Q, \Sigma, \Gamma, \delta, \sqcup, s, F)$ is in state q scanning char c and
u doesn't begin with $\sqcup$
v doesn't end with $\sqcup$
ucv are whole nonblank contents of the tape.
Initial ID
$I_0(x) = [s\, x_1 x_2 \cdots x_n]$.
$[s \sqcup]$ if $x = \epsilon$.

Recall the relation $I \vdash_M J$.

We can write computations as

$[I_0][I_1][I_2][I_3] \cdots [I_t]$

or "ripple style" as

$[I_0]\underbrace{]I_1^R[}\,[I_2]\underbrace{]I_3^R[}\,[I_4] \cdots [I_t]$ reversed if t is odd.

Define (either one can be called ACH_M for "Accepting Computation Histories") Could alternatively say halting

$V_M = \{\vec{c}$ written normally : $\vec{c}$ is a valid finite accepting computation of M on some input $x\}$

$W_M = \{\vec{c}$ written ripple style : $\vec{c}$ is a valid finite accepting computation of M on some input $x\}$

Facts: (much proved in the text) For any M: these rep^ns are computable

- V_M and W_M are both complements of CFLs.
- W_M is the $\cap$ of two DCFLs — One checks $I_j \vdash I_{j+1}$ for j even, the other for j odd. Like Marked Palindrome repeated
- (Green points not in text) V_M can be recognized by a "DFA with 2 Heads".
- $\Rightarrow$ V_M can be recognized by a Linear Bounded Automaton (LBA) — W_M too.
- V_M can be recognized by a "Queue Machine".

OLB

<u>Analysis</u>: $L(M) = \emptyset \iff V_M$ and W_M are empty

And whenever $L(M) \neq \emptyset$, V_M is not a CFL, W_M ditto

<u>Theorem</u>: $\therefore$ $E_{TM} \leq_m$ reduces to all of these problems:

ALL_{CFG} ALL_{NPDA} IE_{DPDA} : INST: DPDAs P_1 and P_2
Ques. Is $L(P_1) \cap L(P_2) = \emptyset$?

EQ_{CFG} is likewise undecidable, not c.e. either.

Reduce ALL_{CFG} $G \hookrightarrow \langle G, G_1 \rangle$ where $L(G_1) = \Sigma^*$
$S \to aS \mid bS \mid \epsilon$.

(Similarly, $ALL_{TM} \leq_m EQ_{TM}$ so EQ_{TM} is neither c.e. nor co-c.e.

$E_{TM} \leq_m E_{DLBA}$, E_{NLBA} — Both DLBA and NLBA are closed under $\sim$, so $\leq_m ALL_{DLBA}$, ALL_{NLBA} too

$NE_{TM} \leq_m \underbrace{NE_{DLBA} \leq_m}_{\text{also}} NE_{2Head\,DFA}$ in fact.

How about ALL_{DPDA}? Decidable <u>since</u> DPDAs can be complemented.

How about ALL_{NFA}? Decidable by NFA $\to$ DFA
but how <u>hard</u> is it? $\hookrightarrow$ Complexity Theory.

Now in Chapter 7

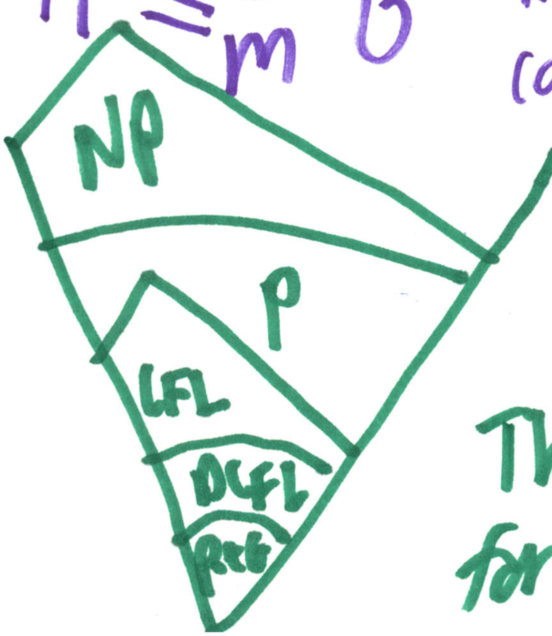
Defn: A TM M runs $\{$ within in time $f(n)$ if for all $x \in \Sigma^*$, taking $n = |x|$, there exists a valid compn $I_0(x), I_1, I_2, \ldots, I_t$ with $t \leq t(n)$ within

If M is nondetc, can demand all valid computations have $\leq t(n)$ steps before halting.

Defn: ① P $\equiv$ denotes the class of languages recognized by DTMs that run within time $p(n)$ for some polynomial p.

② NP $\equiv$ " " by NTMs that run in $p(n)$ time".

③ $A \leq_m^p B$ if there is a polynomial-time computable function $f: \Sigma^* \to \Sigma^*$ s.t. $\forall x \cdot x \in A \iff f(x) \in B$.



CFL $\subseteq$ P by CYK algm.

Thursday will show completeness for NP under $\leq_m^p$ reductions.