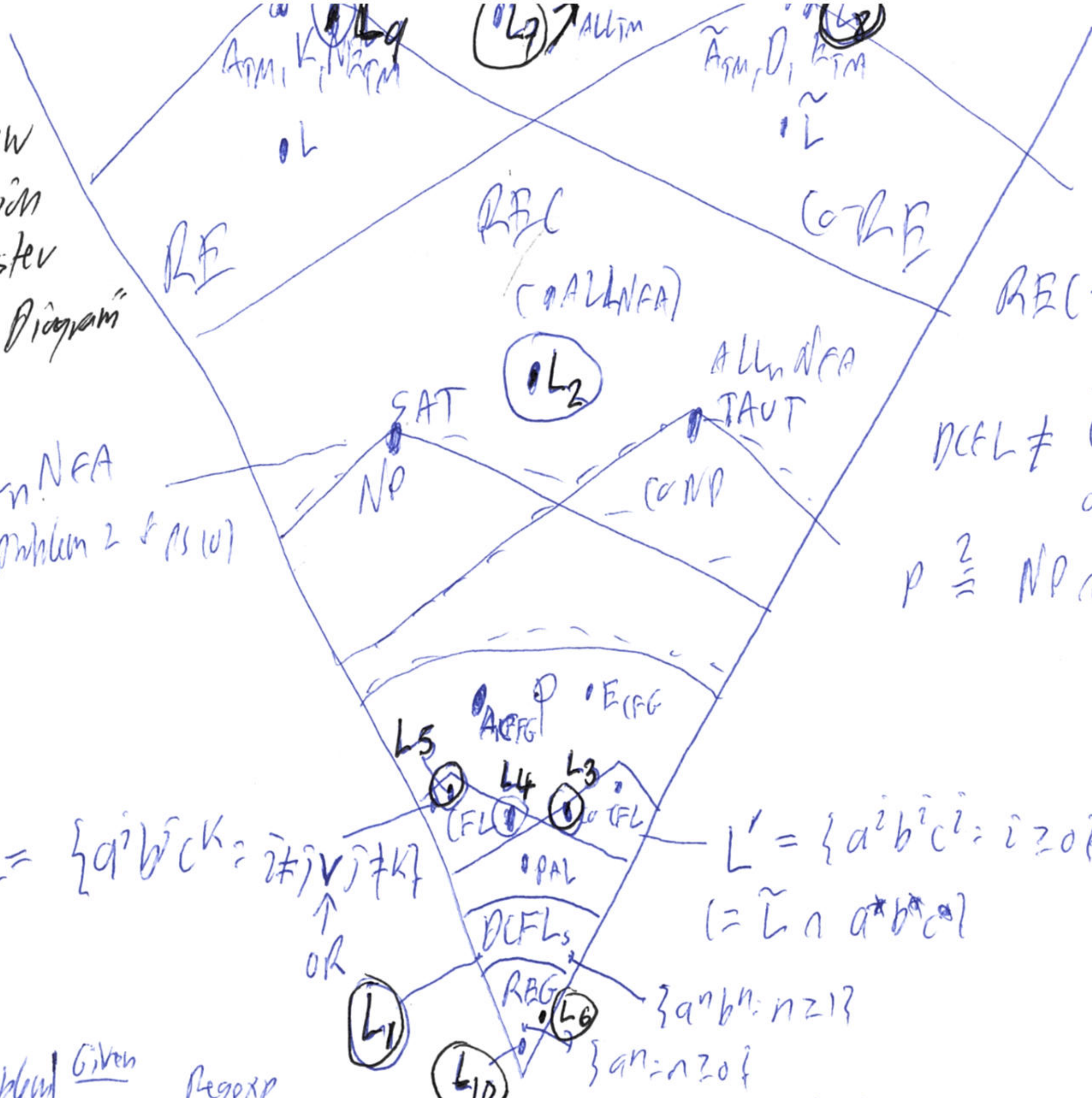
Review Session "Master Cone Diagram"



REC = RE ∩ coRE but DCFL ≠ CFL ∩ co-CFL and P $\overset{?}{=}$ NP ∩ coNP.

NOTALL$_n$NFA (from problem 2 of PS10)

$L = \{a^i b^j c^k : i \neq j \vee j \neq k\}$
↑ OR

$L' = \{a^i b^i c^i : i \geq 0\}$ not a CFL ($= \tilde{L} \cap a^* b^* c^*$)

$\{a^n b^n : n \geq 1\}$

$\{a^n : n \geq 0\}$

Problem Type \ Given	DFA	Regexp NFA	DPDA	CFG or NPDA	DLBA or NLBA	DTM or NTM
"A"					⚠	→ UNDEC O
"E"						
"ALL"		⚠				⊗
EQ		⚠	⚠			⊗
HALT					DEC ←	O
TOT						⊗
DIST						
SUBSET						

$\{M_1, M_2 : L(M_1) \subseteq L(M_2)\}$

"The Undecidability Wall."

O = ce.
⊗ Neither ce nor co-ce
⚠ Decidable but NP-hard.

This is a simpler version of the chart on p5 of my Tue May 3 notes.

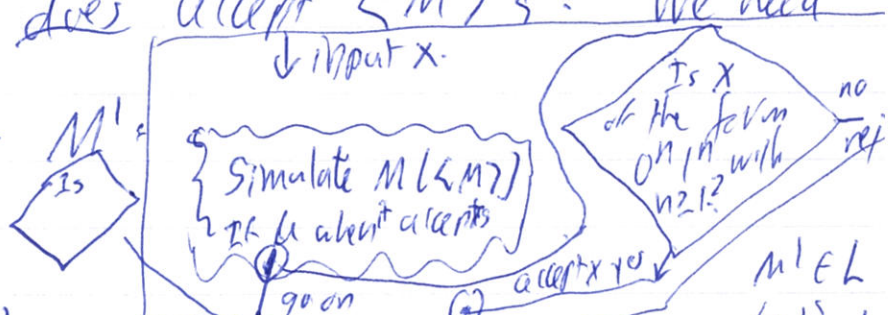
Review Session Answer Versions. PS 10

(1). INST: A_{TM} M ($\Sigma = \{0,1\}$)
QUES: Does there exist a string $x \in \Sigma^*$ s.t. M accepts x and x has the form 0^n1^n for some n

$L = \{\langle M\rangle : L(M) \cap \{0^n1^n : n \ge 1\} \ne \emptyset\}$.

Prove undecidable by reduction from (A_{TM} or) K.
$K = \{\langle M\rangle :$ M does accept $\langle M\rangle\}$. We need:

$M \xrightarrow{f} M'$:



st. $M \in K \iff M' \in L$

i.e. $\langle M\rangle \in K \Rightarrow \forall x\ M'(x)$ goes on $\Rightarrow \forall x\ M'$ acc x iff $x \in A \Rightarrow L(M') = A \Rightarrow \langle M'\rangle \in L$

$\langle M\rangle \notin K \Rightarrow$ for all x, $M'(x)$ does not go on $\Rightarrow L(M') = \emptyset \Rightarrow \langle M'\rangle \notin L$

[L is c.e.: Design an NTM N st. N on input $\langle M'\rangle$ guesses such an x.]

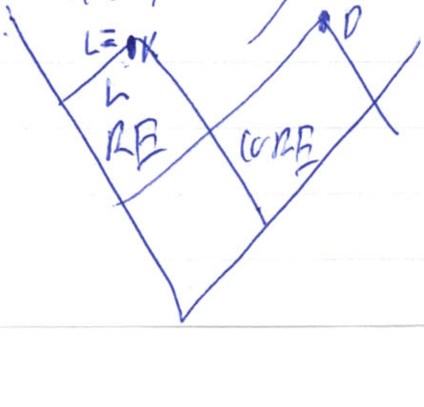
(5): $L' = \{\langle M''\rangle : L(M'') = L(M'')^R\}$. Show undecidable

* $K \le_m \overline{L'}$ by exactly the same construction / reduction fn f, with analysis:

$\langle M\rangle \in K \Rightarrow \dots \Rightarrow L(M') = \{0^n1^n : n \ge 1\}$ which is not equal to its reversal $\Rightarrow \langle M'\rangle \notin L'$, i.e. $\in \overline{L'}$

$\langle M\rangle \notin K \Rightarrow L(M') = \emptyset \Rightarrow \langle M'\rangle \in L'$ since $\emptyset^R = \emptyset$. ∴ $\overline{L'}$

∴ $\overline{L'}$ is undecidable ∴ L' is undecidable.



(despite the text!)

(2) $A_{NFA} \in P$ $\quad A_{NFA} = \{\langle N, x\rangle : N$ is an NFA and N accepts $x\}$.

Notation:

$N = (Q, \Sigma, \delta, s, F)$ $\quad N =$ 

$\underline{m} = \|Q\|$

$\underline{n} = |x|$

The smallest DFA M st. $L(M) = L(N)$ may have up to 2^m states, but we're only going to use $n+1$ of them on input x, so we don't need to build them all ahead of time!

epsilon closure.

We will definitely use the start state $S_0 = E(\{s\})$ of the DFA

• Build S in polynomial time by our general marking alg.

★ At each step i, we have maintained

$$S_{i-1} = \{q : N \text{ can process } x_1 \cdots x_{i-1} \text{ from } s \text{ to } q\}$$

Our algm takes every $q \in S_{i-1}$, processes x_i, and does more ε arcs to build

$$S_i = \{r : N \text{ can process } x_1 \cdots x_i \text{ from } s \text{ to } r\}.$$

Finally your algm accepts $x \Leftrightarrow S_n \cap F \neq \emptyset$. Runtime: $n \cdot$ polynom. = polynomial

Note: even $O(n \cdot m^4)$ is polynomial in the length of $|\langle N, x\rangle| \approx m + n$.

★ FACT: $A_{CFG} \in P$ too – ~~applies Bynam~~

③ INST: $\langle N, n \rangle$ where N is an NFA, n is a number ($n \leq |Q|$)
QUES: Does N accept all strings of length n?

if word were "any" $L^* = \{\langle N, n\rangle : L(N) \cap \{0,1\}^n \neq \emptyset\}$

$L = \{\langle N, n \rangle : \{0,1\}^n \subseteq L(N)\}$

$\tilde{L} = \{\langle N, n \rangle : \{0,1\}^n \not\subseteq L(N)\}$

$= \{\langle N, n \rangle :$ there exists a string $x \in \{0,1\}^n$ that N does not accept$\}$

Think of an NTM N'

guess x: x is shorter than the input $\langle N, n \rangle$.

verify $\langle N, x \rangle \notin A_{NFA}$

Since A_{NFA} is in P and P is closed under ~, $\tilde{A}_{NFA}$ is in P too.

Hence N' has the form [guess shorter] [poly-time body]
So N' is a polynomial-time NTM
or say the body is a poly-time verifier for $\tilde{L}$.

Either way, $\tilde{L} \in NP$.

Part (c) was an example of manipulating NFAs.