"3n+1 Binary" 
By K.W. Regan 
Illustrates the "Collatz Problem."

Start with a binary number \( n \) on the tape, and position the tape head anywhere to the left of \( n \).

Erase any leading 0s in \( n \), skip over leading blanks.

This Turing machine \( M \) begins with a number \( n \geq 1 \) written in binary on its tape. If \( n=1 \), the machine accepts. If \( n \) is even, \( M \) divides it by 2. If \( n \) is odd, \( M \) multiplies \( n \) by 3 and adds 1.

This process continues, and halts if and only if \( n \) becomes 1.

Mathematicians believe that every number \( n \geq 1 \) eventually becomes 1, but no one has proved it yet. Hence here is a simple Turing machine for which nobody knows whether it halts for all inputs, or not! It is not even known whether \( L(M) \) is recursive, although of course \( L(M) \) is believed to be all of \( \mathbb{N} \).