Name and St.ID#:____

CSE439, Fall 2025 — Prelim I — Oct. 9, 2025

Open book, open notes, closed neighbors, 75 minutes. The exam totals 100 pts., subdivided as shown. Do *all five problems* on these exam sheets—there is no "choice" option. *Show your work*—this may help for partial credit.

Use of a quantum circuit simulation app—whether online or downloaded—or matrix calculator is forbidden. You may use—and cite—theorems and facts from lectures and homeworks without further justification.

Notation: You may freely mix "standard linear algebra notation" and "Dirac *bra-ket* notation" for quantum states. For example, e_{10} means the same thing as $|10\rangle$ and is represented (in big-endian notation) by the unit column vector $[0, 0, 1, 0]^T$. The vectors $|+\rangle = \frac{1}{\sqrt{2}}[1, 1]^T$ and $|-\rangle = \frac{1}{\sqrt{2}}[1, -1]^T$ really don't have good "non-Dirac" names, however, and for matrix outer-products the *ket-bra* form typified by $|+\rangle\langle-|$ is standard. Quantum state vectors use big-endian notation by default; if you switch to little-endian you must say so.

(1) (3+3+3+7 = 16 pts.) *Unit(ary) vectors and matrices.*

Calculate—or otherwise build—vectors or matrices meeting the respective conditions.

(a) Vector $|-\rangle \otimes |1\rangle$.

(b) A 2×2 unitary matrix U such that $U|-\rangle = |1\rangle$.

(c) The outer product $|-\rangle\langle 1|$.

(d) A separable 4×4 matrix V such that $V[0, 1, 0, -1]^T = [1, 1, 0, 0]^T$. (If you work out 2×2 matrices A and B such that $V = A \otimes B$ in advance, then you don't need to multiply them out, or do any 4×4 matrix computations at all.)

(a) ___________ (b) __________ (c) ___________ (d) ___________________

(2) (18 pts. total) *True/False.*

Please write out the words `true` and/or `false` in full, in the blanks below or clearly next to the statements. A justification is **required** for part (e) and worth 3 points by itself.

(a) If $f(n) \sim 2\log_2 n)$ then $2^{f(n)} = \Theta(n^2)$.

(b) If U and V are 2×2 unitary matrices, then $U \cdot V = V \cdot U$.

(c) It is possible for a 4×4 unitary matrix U to have $U\,|00\rangle = |00\rangle$, $U\,|01\rangle = |11\rangle$, and $U(|0+\rangle = |{+}{+}\rangle)$, where $|0+\rangle$ means $|0\rangle \otimes |+\rangle$.

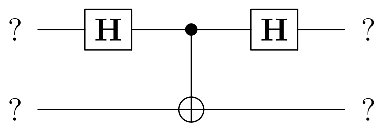
(d) It is possible for an n-qubit unitary matrix U to make $U\,|+^n\rangle = \mathbf{0}$, where $\mathbf{0}$ is the all-zero vector of length $N = 2^n$.

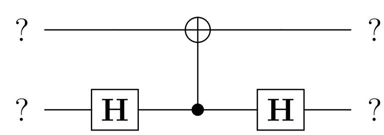
(e) It is possible to design a 2-qubit quantum gate (or circuit) U such that on the four standard basis states, $U\,|ab\rangle$ flips bit b if and only if both bits are 1, else it does nothing.

`(a) _________   (b) _________   (c) _________   (d) _________   (e) _________`

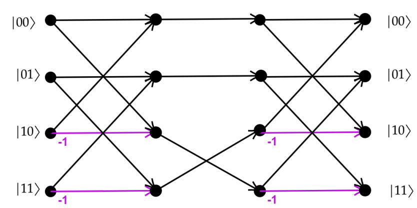
(3) (9 + 12 = 21 pts.)

Consider the following two two-qubit quantum circuits, in which the second has an "upside-down CNOT gate":

$C_1 =$ 

$C_2 =$ 

We have basically seen the maze diagram for the first circuit C_1 in lectures: it is



(a) Draw the maze diagram of the second circuit, C_2.

(b) Prove that the two circuits C_1 and C_2 are equivalent—i.e., that for every two-qubit quantum state $\mathbf{\Phi}$, $C_1\mathbf{\Phi} = C_2\mathbf{\Phi}$. There are several fine ways to do so: multiply out the 4×4 matrices, show the same respective actions on the four standard basis states, or show that the two maze diagrams have equivalent paths between each of the four source nodes at left and each of the four target nodes at right.

(4) (24 pts. total)

For each of the following two-qubit quantum state vectors or matrices, answer whether it is separable. If you say yes, write it as the tensor product of two single-qubit vectors-or-matrices; if no, prove your answer via equations or other reasoning.

(a) $\frac{1}{2}[-1, 1, 1, 1]^T$.

(b) $\frac{1}{2}[-1, -1, i, i]^T$.

(c) $(\mathbf{S} \otimes \mathbf{Z}) \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot (\mathbf{Z} \otimes \mathbf{S})$.

(5) (21 pts. total—end of exam)

With reference to problem (1) of Assignment 3, let "$(-1)^{OR}$" stand for the operation that leaves $|00\rangle$ alone but negates the phase of the other three standard basis states.

(a) Design a 2-qubit quantum circuit C out of basic gates that executes this operation. Give some explanation of how and why your circuit works.

(b) Also give the value of C on input $\frac{1}{\sqrt{2}}[0, 1, 0, -1]^T$ and say whether C changed that state.