CSE439, Fall 2026 **Problem Set 2** **Due Fri. 9/25, 11:59pm**

Reading: For next week, read the rest of Chapter 5, read Chapter 6, and yes also read Chapter 7. You may find that Chapter 7 actually recapitulates a lot of stuff; for instance, I have already made the boxed point in section 7.6 about entanglement. Also read the Chapter 7 end notes, and for Yogi Berra, see https://yogiberramuseum.org/about-yogi/yogisms/.

——————-*Assignment 2, due Fri. 9/25 "midnight stretchy" on CSE Autolab*——————-

(1) (a) First, a little bit of matrix practice. Calculate the 4×4 matrix E of a two-qubit quantum circuit that begins with one Hadamard gate on line 1, then a CNOT gate with control on 1 and target on 2 (so far, the basic entangling circuit shown in lecture), and then has one more Hadamard gate on line 1, after the CNOT gate. Also answer and justify, is E a separable matrix? (12 pts. total)

(b) Now for a theoretical question: Suppose we are given a 4×4 matrix E that can be written as a sum of two separable matrices. That is, $E = A \otimes B + C \otimes D$ where A, B, C, D are all 2×2 matrices. And suppose we apply E to a separable column vector $w = u \otimes v$, for instance $[1, 2, 3, 6]^T = [1, 3]^T \otimes [1, 2]^T$ also from lecture. By linearity, we can calculate $x = Ew$ using only 2×2 martrix operations via:

$$x = (A \otimes B)(u \otimes v) + (C \otimes D)(u \otimes v) = (Au \otimes Bv) + (Cu \otimes Dv).$$

The *question*: is the vector x always separable? Try this with your matrix E in part (a). (18 pts. for a proof that x is always separable, if so, or else finding suitable A, B, C, D for the above E, doing the calculation $x = Ew$ using A, B, C, D as above, checking Ew directly, and telling whether x is separable. This makes 30 total on the problem.)

(2) (a) Design a 3×3 matrix U in which exactly one entry is a 0, such that U is both unitary and Hermitian. You can do so without complex numbers, so "Hermitian" just means the matrix is symmetric.

(b) Do likewise for a 3×3 matrix V in which exactly four entries are 0.

(c) Prove that there are no 3×3 unitary matrices in which exactly two or exactly three entries are 0, even if complex numbers are allowed (and even if the matrix need not be Hermitian too). ($12 + 6 + 12 = 30$ pts.)

(3) Design three-qubit quantum circuits that given the all-zero basis state e_{000} as input create the states

(a) $\frac{1}{2}(e_{000} + e_{100} + e_{010} - e_{111})$,

(b) $\frac{1}{2}(e_{000} + e_{100} - e_{010} - e_{111})$, and

(c) $\frac{1}{2}(e_{000} - e_{110} + e_{001} + e_{111})$.

You may experiment and check your work with a quantum circuit simulator. Recall that multiplying everything by any unit scalar, in particular -1, gives the same quantum state. For a warmup, note that the 2-qubit circuit `H 1 H 2 CZ 1 2` applied to e_{00} produces the state $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$. This is plain-text notation for the circuit with Hadamard gates on qubits 1 and 2 followed by a **CZ** gate between them. You are also welcome to use Dirac notation, so that e.g. the state in (c) is written $\frac{1}{2}\left(|000\rangle - |110\rangle + |001\rangle + |111\rangle\right)$. The circuit in (c) can be a graph state circuit, with **CZ** and **Z** gate(s) between an initial and final $\mathbf{H}^{\otimes 3}$. $(9 + 12 + 9 = 30$ pts., for 90 pts. total on the set)