Open book, open notes, closed neighbors, 48 minutes. The exam totals 60 pts., subdivided as shown. Show all work—this may help for partial credit. It is AOK to do the problems out-of-order (shorter ones first). All notation is standard as in course readings and lectures.

The submission logistics are the same as for homeworks. Please include in your submission a signed statement that this represents my own work in accordance with University regulations.

(1) (18 + 3 + 3 + 9 = 33 pts.)

The following nondeterministic finite automaton $N = (Q, \Sigma, \delta, s, F)$ has $Q = \{1, 2, 3\}$, $\Sigma = \{a, b\}$, $s = 1$, $F = \{1\}$, and $\delta = \{(1, a, 3), (1, b, 2), (2, a, 1), (2, b, 2), (2, \epsilon, 3), (3, a, 3), (3, b, 1)\}$.

![NFA Diagram]

(a) Use the NFA-to-DFA algorithm to convert $N$ into a DFA $M$ such that $L(M) = L(N)$.

(b) Does there exist a “dead string” $x \in \Sigma^*$, i.e., such that for all $z \in \Sigma^*$, $xz \notin L(M)$?

(c) Does there exist an “eternal string” $y \in \Sigma^*$, i.e., such that for all $z \in \Sigma^*$, $yz \in L(M)$?

(d) Calculate a 2-state generalized NFA $G$ such that $L(G) = L(N)$. Note that you are only being asked to eliminate one state—you don’t have to give the final regular expression or go all the way to a 1-state machine. It is your choice of which state to eliminate.

(2) (18 pts.)

Over $\Sigma = \{a, b\}$, define $L = \{vbw : \#a(v) = \#b(w), v, w \in \Sigma^*\}$. Prove using the Myhill-Nerode technique that $L$ is not a regular language.

(3) (9 pts. total)

(a) If $M$ is a DFA and $N$ is an NFA, then is the language $L(M) \setminus L(N)$ always decidable?

(b) If $M$ is a DFA and $N$ is a nondeterministic Turing machine, then is the language $L(M) \setminus L(N)$ always decidable?

Justify your answers briefly.

End of Exam.