CSE491/596, Fall 2020 Second Prelim Exam Nov. 30, 2020

Open book, open notes, closed neighbors, 48 minutes. The exam totals 60 pts., subdivided as shown. *Show all work*—this may help for partial credit. All notation is standard as in course readings and lectures. The second problem has a choice, (2a) XOR (2b). You may freely cite facts from lectures and notes, such as the languages A_{TM}, K_{TM}, and NE_{TM} being (c.e. but) undecidable and the languages D_{TM} and E_{TM} being (co-c.e. but) undecidable.

The submission logistics are the same as for homeworks. Please include in your submission a signed statement that *this represents my own work in accordance with University regulations.*

(1) (18 + 6 + 6 = 30 pts.)

The following two decision problems involve directed graphs G in which every non-sink node has two out-edges labeled "right" and "left." Also specified in an instance are a source node s, a sink t, and another node u.

(i) INSTANCE: G, s, t, u as above.
QUESTION: Is there a path from s to t that goes through u, such that the number of steps from s to u equals the number of steps from u to t?

(ii) INSTANCE: G, s, t, u as above.
QUESTION: Is there a path from s to t that goes through u, such that the sequence of "right" and "left" edges taken by the path from s to u equals the sequence from u to t?

Note that problem (ii) involves a more-particular question, since in order for the right-left sequences to be the same, they must have the same number of steps.

(a) Show that problem (i) belongs to P, indeed to NL. A pseudocode or machine sketch that is detailed enough to show time and/or space usage is sufficient. (15 pts.)

(b) Briefly explain why your argument in (a) fails to classify problem (ii) the same way. (6 pts.)

(c) The second problem is in fact complete for one of NP or co-NP. Say and briefly justify which of those two classes it **belongs to**. (6 pts.; do not try to prove completeness)

(2) (30 pts. total)

Choice—Do EXACTLY ONE of the following two problems, (2a) XOR (2b). You must indicate clearly which you are attempting. The problems are on the next page.

(2a) Consider the following decision problem:

NEVERHALT

INSTANCE: A deterministic Turing machine $M = (Q, \Sigma, \Gamma, \delta, \_, s, F)$.

QUESTION: Is it the case that, for all inputs $x \in \Sigma^*$, $M(x)$ never halts?

(i) Prove by a mapping reduction that the language of this problem is not c.e.

(ii) Is the language co-c.e.? Justify your answer.

XOR

(2b) Define a "star" in a directed graph G to be a node u together with all nodes v such that u has an edge to v. Prove that the language of the following decision problem is NP-complete, using a polynomial-time mapping reduction from 3SAT for the hardness part.

STAR COVER

INSTANCE: A directed graph $G = (V, E)$ and a natural number k.

QUESTION: Can V be written as a union of k or fewer stars?

For a note and hints, a directed graph is allowed to have edges from u to v and from v to u; you may use this idea in the "rungs." You may also treat the whole graph as undirected—unlike in HW6, problem (1), the directed edges make the proof easier to argue but are not critical. Your reduction should have sections labeled *Construction*, *Complexity*, and *Correctness*.

END OF EXAM.