CSE491/596, Fall 2021 Problem Set 6 (the last) Due Fri. Dec. 10

The **Final Exam** is scheduled for **Friday, Dec. 17, 11:45am–2:45pm**. It will be held in person in the lecture room, **120 Clemens**, unless UB is forced to go remote. It will have the same rules and in-person logistics (with registered exception) as the prelim exams; the final in this course has always been open-book, open-notes.

Lectures and Reading:

Same as stated before: posted excerpts from my joint textbook *Introduction to Quantum Algorithms Via Linear Algebra.* They are:

- https://cse.buffalo.edu/~regan/cse491596/LRQmitbook2pp3-106.pdf up through ch. 8;
- https://cse.buffalo.edu/~regan/cse491596/LRQmitbook2pp131-147.pdf up through section 14.4.

———- *Assignment 6, due Dec. 10 "midnight stretchy" on CSE Autograder* ———-

(1) (24 pts. total)

Define L to be the language of Boolean formulas ϕ such that either:

- ϕ is in conjunctive normal form and is satisfiable, or
- ϕ is in *disjunctive normal form* (DNF) and is a tautology.

DNF means $\phi = T_1 \lor T_2 \lor \cdots \lor T_m$ where each *term* T_j is a conjunction of literals. (You may if you wish limit each term to at most three literals, like how 3SAT does for CNF.)

(a) Show that if L is in NP, or if L is in $\mathsf{co\text{-}NP}$, then $\mathsf{NP} = \mathsf{co\text{-}NP}$.

(b) Show that $L \leq_m^p$ TQBF, so that $L \in \mathsf{PSPACE}$ follows. Also briefly sketch how L is in linear space directly.

(2) (12 pts.)

Recall the definition of *Halves*(L) from the previous assignment. Suppose it were true that for every $L \in \mathsf{P}$, the language *Halves*(L) is in RP. Prove how it would follow that $\mathsf{NP} = \mathsf{RP}$.

(3) 60 pts. Five short quantum problems (mostly linear algebra), each worth 12 pts. The ones from the Lipton-Regan text refer to page/problem numbers in the above excerpts.

(a) Exercise 3.10 on page 27. Also show $\mathbf{V} = \mathbf{HSH}$ and square $\mathbf{HSH}$ to check your answer.

(b) Exercise 3.16 on page 28, using the trig identities for $\cos(2\theta)$ to verify the second part.

(c) Exercise 5.11 on page 55, for 4×4 matrices, where $\mathbf{J}$ is the matrix with every entry 1/4.

(d) Calculate the 4×4 matrix $\mathbf{E}$ of the operation computed by the two-qubit quantum circuit formed by placing a Hadamard gate on qubit line 1, then a **CNOT** gate with control on 1 and target on 2, then another Hadamard gate on line 1. In matrix-product terms, this is $(\mathbf{H} \otimes \mathbf{I}) \cdot \mathbf{CNOT} \cdot (\mathbf{H} \otimes \mathbf{I})$.

Then show that the same matrix results from flipping the circuit upside-down—that is, having **H** on line 2, then **CNOT 2 1** meaning with control on line 2 and target on line 1, followed by **H** on line 2 again. Use a quantum circuit web applet such as https://wybiral.github.io/quantum/ or *Quirk* to check this.

(e) Exercise 6.6 on page 70—note the connection to exercise 3.16 in part (b).

This makes 96 regular-credit points on the set. An extra-credit option has been posted separately.