Open book, open notes, closed neighbors, 48 minutes. The exam totals 80 pts., subdivided as shown. Show all work—this may help for partial credit. Please do the problems on these exam sheets.

All notation is standard as in course readings and lectures; in particular, the reversal of string $x$ is written $x^R$, with examples $011^R = 110$, $\epsilon^R = \epsilon$, and $x = x^R$ means $x$ is a palindrome. The difference of two sets $B$ and $A$ is written $B \setminus A$; it also equals $B \cap \bar{A}$ where $\bar{A}$ stands for the complement of $A$.

(1) (24 pts.)

Over $\Sigma = \{0, 1\}$, define $L$ to be the language of strings that have more occurrences of the substring 010 than occurrences of the substring 101. Prove using the Myhill-Nerode technique that $L$ is not a regular language. (Note that occurrences of the substrings can overlap—but you can take that worry out of the picture by judicious choices of $S$ and/or $z$.)
Consider the regular expression \( r = (aa + ba)^*(ab + bb)^* \) over the alphabet \( \Sigma = \{a, b\} \).

(a) Design an NFA \( N \) with 4 states such that \( L(N) = L(r) \).

(b) Design a DFA \( M \) such that \( L(M) = L(r) \). You may either convert your NFA \( N \) from part (a) or design \( M \) by understanding \( r \) directly, but in the latter case you must have comments that explain the strategy. In particular, you must have at least one comment saying when the \((ab + bb)^*\) part comes into play.

(c) Does there exist a “kill string” \( u \) such that for all \( z \in \Sigma^* \), \( uz \notin L(r) \)? Give a shortest such string if so.

(d) Convert your \( N \) into a 2-state generalized NFA \( G \) such that \( L(G) = L(N) \). You need not follow the algorithm but can “just do it”—and this is worth 2 points mainly because 80 is congruent to 2 modulo 3.
(Space for Problem (2), continued...the last problem is overleaf)
(3) **(24 pts. total)** True/False with justifications: each question is 3 pts. for correct answer and 3 for an explanation (or definitive example or counterexample) in one or two sentences.

(a) If $x$ is any string in the language $L(r)$ in problem (2), then its reversal $x^R$ is also in $L(r)$.

(b) Every undecidable language is non-regular.

(c) If $B$ is undecidable and $A$ is decidable, then $B \setminus A$ must be undecidable.

(d) If $B$ is non-regular and $A$ is regular, *and* if $A \subset B$, then $B \setminus A$ must be non-regular.

**End of Exam.**