Name and St.ID#:____

CSE491/596, Fall 2022 Prelim II Nov. 30, 2022

Open book, open notes, closed neighbors, 48 minutes. The exam totals 80 pts., subdivided as shown. The third problem has a *choice*, (3a) XOR (3b); youmay attempt only one and must specfity clearly which you are attempting. Do *all three problems* on these exam sheets. *Show your work*—this may help for partial credit.

Notation: The complement of a language A is denoted by $\tilde{A}$ and equals $\Sigma^* \setminus A$, where $\setminus$ means difference of sets. Given sets A and B, the *symmetric difference* $A \bigtriangleup B$ is the same as $(A \cap \tilde{B}) \cup (B \cap \tilde{A})$, and also the same as $(A \setminus B) \cup (B \setminus A)$. In the way that union is like OR, it corresponds to the logical exclusive-or (XOR) operation.

Also given a language A, its *reversal* is $A^R = \{x^R : x \in A\}$. Note that A can contain strings x that are not palindromes (i.e., such that $x \neq x^R$) and still satisfy $A = A^R$, provided A also has x^R.

(1) (5 × 4 = 20 pts. total) *True/False/Unknown*. Please write out the words `true`, `false`, and/or `unknown` in full. Brief justifications are not necessary but may help for partial credit—especially between the answers `false` and `unknown`.

(a) If $A = A^R$ and B is the complement of A, then $B = B^R$.

(b) If $A \leq_m^p$ SAT and B is the complement of A, then $B \leq_m^p$ SAT.

(c) There is a computable time function $t(n)$ such that every decidable language can be decided in time $O(t(n))$.

(d) Every language in $\mathsf{NTIME}[O(n^2)]$ belongs to PSPACE.

(e) If $A \in \mathsf{NP}$ and $B \in$ co-NP, then $A \setminus B$ belongs to NP.

(2) 30 pts. total

Consider the following decision problem:

LOCKSTEP EDGE-DISJOINT PATHS
INSTANCE: A directed graph $G = (V, E)$ and nodes $s_1, s_2, t_1, t_2 \in V$.
QUESTION: Do there exist a path $(s_1 = u_0, u_1, u_2, \ldots, u_{r-1}, u_r = t_1)$ and a path $(s_2 = v_0, v_1, v_2, \ldots, v_{r-1}, v_r = t_2)$ such that for all steps $i = 0$ to r, no edge connects the vertex u_i used at step i in the first path to the vertex v_i used at step i in the second path?

Unlike with EDGE-DISJOINT PATHS, edges between vertices at different timesteps in the paths are allowed. **Show that this problem belongs to P.** Here are two suggested ways to do so—either one is fine:

- Construct a directed graph $G' = (V', E')$ whose vertices are *certain pairs of nodes* in V, and whose edges are [...you define them too—formal definitions of V' and E' are required for full credit]. Then apply to G' a polynomial-time algorithm we have seen in the course. Or:
- Show how to build a nondeterministic Turing machine N that accepts this LOCKSTEP EDP language in $O(\log n)$ space, where $n = |V|$ as usual. It is enough to sketch the tapes of N and say what each one does—including why the read-only input tape holding G (in either matrix or edge-list form, your choice) can be navigated using the $O(\log n)$-sized worktapes; no instruction arcs need to be given. Then use a theorem in the course to reach your conclusion.

[Technotes—you may ignore these: You could consider G to have self-loops at the terminal nodes t_1 and/or t_2. This has the same effect as not requiring the length r of the two paths to be the same—this works out OK so long as the last nodes of the longer path do not have edges to the terminal node of the shorter path. No essential generality is lost by restricting G to be acyclic (i.e., a DAG) with sinks t_1 and t_2 (except allowing said self-loops at those sinks).]

(2) (workspace)

(3) (30 pts. total)

Choice—Do Exactly One of the following two problems, (3a) XOR (3b). You must indicate clearly which you are attempting. Your reductions in either case should have sections labeled *Construction*, *Computability/Complexity*, and *Correctness*.

(3a) Define B to be the language of (codes $\langle M \rangle$ of) deterministic Turing machines M such that for some string x, M accepts x but does not accept x^R.

(i) Prove by a mapping reduction that B is undecidable. (18 pts.)

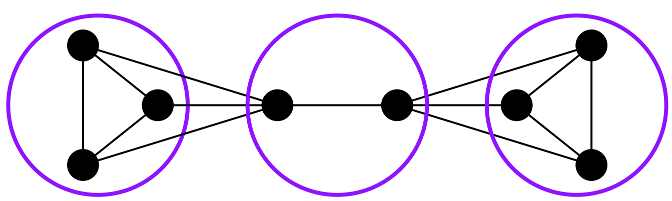
(ii) Is B c.e.? If you say yes, give a formal definition of B using quantifiers and decidable predicates that justifies your answer. If not, sketch another reduction if your answer to (i) did not already have this consequence. (12 pts.)

XOR

(3b) Prove that the language of the following decision problem is NP-complete, using a polynomial-time mapping reduction from 3SAT for the hardness part:

Associations
Instance: An undirected graph $G = (V, E)$ where V is given as a union of some number k of disjoint subsets, $V = S_1 \cup S_2 \cup \cdots \cup S_k$.
Question: Is it possible to choose one vertex u_i from each subset S_i such that each chosen u_i is connected by an edge to at least one other chosen vertex u_j?

For example, the following graph with $n = 8$ vertices and $k = 3$ subsets (shown via circles) is an instance where the answer is **no**, because whichever node you choose from the middle subset, one of the chosen nodes on the ends will be isolated.



(3) (workspace)

End of Exam