CSE491/596, Fall 2023 Problem Set 1 Due Mon. 9/18, 11:59pm

Lectures will be undisturbed until Friday Sep. 22 and Monday Sep. 25. Current plan is to try to make those up on the Wednesday evenings before and/or after then. Homeworks will eventually be due on Friday evenings, but this one is due Monday the 18th and it is unclear what day works best for the second assignment.

My regular office hours are fixed as **Tuesdays 1–3pm** and **Fridays 11am–noon** and **3–4pm** (i.e., after class). This takes effect next week.

Lectures and Reading

Week 3 will complete the proof of *Kleene's theorem* giving the conversions among regular expressions, NFAs, and DFAs. It will also cover the *Cartesian product construction* of DFAs and what it says about operations that the class of regular languages is closed under. Please read the brief treatments in the notes by Arun Debray and other materials on the course webpage. Then look ahead to the coverage in the posted notes from last year's Week 3 *and* the first half of the Week 4 Monday lecture.

The following asssignment is for submission *as a single PDF file* to **CSE Autograder**, which will be set up to receive it in a few days. The deadline is "midnight stretchy"; there is an overnight grace period without penalty. Your submission must be *your own individual work*, subject to conditions that were discussed in class on Friday 9/8.

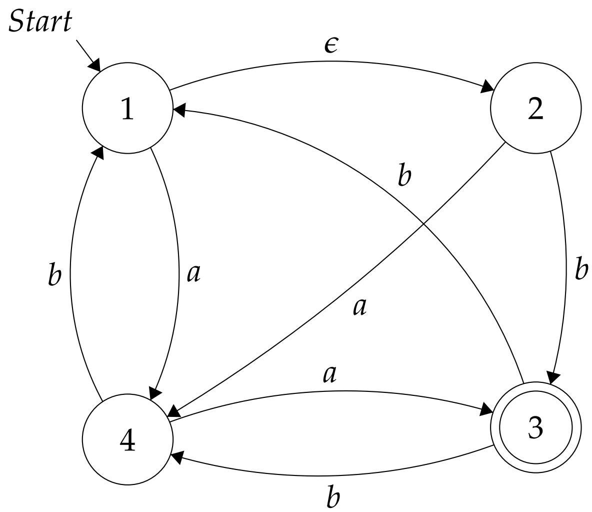
(1) Consider the "spears and dragons" DFA from the demo, which had alphabet $\Sigma = \{\$, D, 0\}$ where '0' means an empty room. Build deterministic finite automata that model the following alterations to the game:

(a) You may hold up to 2 spears, but not 3. If you have 2 spears and kill a dragon, you are down to one spear.

(b) You may hold up to 2 spears, but may only hold 2 spears for a limited time. If you get two empty rooms after picking up the second spear, you have to drop down to carrying one spear before entering the next room. If you get an empty room and then another spear, or two more rooms with spears, the interpretation is that you are "refreshed" and can carry two spears through another empty room. If you get another spear and then an empty room, you will have to drop a spear if the room after that is empty.

You are required to write comments on each state and on some important transitions that explain how your intrepretation of the rules is executed. In each case, the language of your machine should be the set of strings $x \in \Sigma^*$ that represent "dungeons" where the Player exits without being killed by a 'D'ragon. It is fine for the Player to exit holding zero spears, one spear, or two spears. (Points are 6 for (a) and 18 for (b).)

(2) Convert the followjng NFA into an equivalent DFA. The components of the NFA are $\Sigma = \{a, b\}$, $Q = \{1, 2, 3, 4\}$, $s = 1$, $F = \{3\}$, and $\delta = \{(1, \epsilon, 2), (1, a, 4), (2, a, 4), (2, b, 3), (3, b, 1), (3, b, 4), (4, a, 3), (4, b, 1)\}$.



Also answer the following questions:

(a) Find two strings x, x' of the shortest possible length(s) such that for each of its four states q, N can process x from s to q, and likewise x'.

(b) Find the shortest string that N cannot process from s at all.

(c) Is there a string y such that regardless of what state q N starts in, N cannot process y?

You may find your DFA most helpful to answer those questions with. (Points are 18 for the DFA and 6+3+3 for the questions, making 30.)

(3) Calculate a regular expression over $\Sigma = \{a, b\}$ for the language of strings that are **not** accepted by the following NFA: $Q = \{s, q, f\}$, $F = \{f\}$, and

$$\delta = \{(s, a, q), (s, b, f), (q, b, s), (q, a, f), (f, a, s), (f, \epsilon, q)\}.$$

(Note that if the last instruction were on b not ϵ it would be a DFA.) You must use a strategy based on theorems in lectures and posted notes, not just inspection (that is, "hacking"). However, for the last stage, you need not execute the formal (D)FA-to-regular expression algorithm covered later in the week; in this case, there are ways to express what the algorithm does more directly. (24 pts., for 78 total on the set.)