CSE491/596, Fall 2023: Demo of the "Turing Kit" and NFAs

[Most of this lecture will be a demo of the "Turing Kit" software on the course webpage. This will not be required for any assignment, just optional. The demo will include a look-ahead to Turing machines.

- If the demo glitches, we will try again Friday and give the Friday Week 1 lecture now.
- If time allows, NFAs will be introduced now; else they come in next Wednesday's lecture.

The first assignment will be given out in Week 2.]

One usefulness of the "set of triples" definition of a DFA is that it extends naturally to define a nondeterministic finite automaton (NFA). We can recap it in a general way:

The formal definition of a **finite automaton** is a 5-tuple (i.e., an object) $N = (Q, \Sigma, \delta, s, F)$ where:

- Q is a finite set of *states* `set<State> Q;`
- Σ is the *input alphabe*t `set<char> Sigma;`
- s, a member of Q, is the *start state* (also called q_0) `State s;`
- F, a subset of Q, is the set of *accepting* states (also called *final* states) `set<State> F;`
- δ is a finite set of *instructions* (also called *transitions*) of the form (p, c, q) where $p, q \in Q$ and $c \in \Sigma$.
  `set<Triple<State,char,State> > delta;`

The machine is **deterministic** (a **DFA**) if $(\forall p \in Q)(\forall c \in \Sigma)(\exists! q \in Q) : (p, c, q) \in \delta$. Else it is "properly" **nondeterministic** (an **NFA**).

So DFA is a special case of an NFA. When we have a DFA M, we can regard δ as a function from $Q \times \Sigma$ to Q. With an NFA, we could regard δ as a function from $Q \times \Sigma$ to 2^Q, which is the set of all subsets of Q and called the *power set* of Q. But in most cases I prefer to think of δ as a set of instructions.

NFAs with ϵ-transitions

The NFA can be augmented by allowing it to move from a state p to a state q without changing a character. The instruction is then written as (p, ϵ, q). Then we have

$$\delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times Q.$$

This is confusing because ϵ is a string, not a character in Σ. We will shortly see the point of this extension. The Sipser text, on which Debray's notes are based, makes this the standard definition of NFA, while other sources call it an NFA with ϵ-transitions (**NFA**$_\epsilon$). The extension does not affect the formal definition of the language of the NFA, which I give in my own terms as follows:

Say that N ***can process*** a string x ***from*** state p ***to*** state q if there is a sequence of instructions

$$(p, c_1, q_1)(q_1, c_2, q_2)(q_2, c_3, q_3) \cdots (q_{m-2}, c_{m-1}, q_{m-1})(q_{m-1}, c_m, q)$$

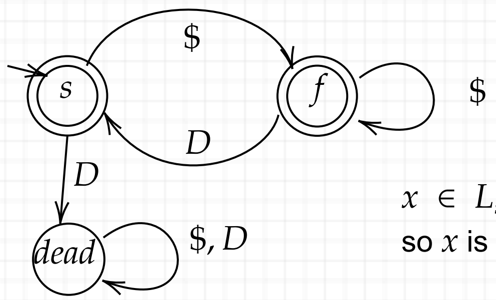
such that $c_1 c_2 \cdots c_m = x$. Then we write $x \in L_{pq}$ (with N understood). Now formally define:

$$L(N) = \cup_{f \in F} L_{sf}.$$

If N has only one accepting state f (a design goal we can meet for NFAs but often not for DFAs) then the language is just L_{sf}. We will find the L_{pq} concept especially handy with "GNFAs" later on.

$x = \$DD$

$N:$



$x \in L_{s,dead}$ but $dead$ is not accepting so x is not in the language.

Without the dead state and arc to it, the NFA N on input $x = \$DD$ would "crash" in state s. Even though s is an accepting state (and even though this would count as legal termination by a Turing machine), not all of x would be processed, so it does not count in the FA's language. With the dead state present, x gets processed to $dead$, but $dead \notin F$ so $x \notin L(N)$ still.