Myhill-Nerode Examples (Wed Sept. 23 after console crashed)

$L_0 = \{ ah^nb : n \geq 0 \}$. Take $S = \{ an: n \geq 0 \} = a^*$. Clearly $S$ is infinite.

Let any $x \in S$, $x \neq \epsilon$ be given. Then we can write

$X = a^m$, $Y = a^n$, where $m+n$

Take $Z = b^m$. Then $XZ = a^m b^m \in L_0$

But $YZ = a^n b^m \notin L_0$ because $n \neq m$.

Hence $S$ is PD for $L_0$, and since $S$ is infinite, $L_0$ is not regular by MNJ.

$L_1 = \{ \emptyset D^n : n \geq 0 \}$. Take $S = \emptyset^*$, clearly infinite.

Let any $x \in S$ be given. Then $X = \emptyset^m$, $Y = \emptyset^n$ where wlog. $m < n$.

Take $Z = D^n$? Then $XZ = \emptyset^m D^n \notin L_0$

Hence we do not need to care

$L_2 = \{ x \in \{a,b\}^* : x \text{ is a palindrome, i.e. } x = x^R \}$. Examples:

$abba \in \{a,b\}$; $baba$ not a palindrome.

Take $S = a^* b$. Let any $x \in S$, $x \neq \epsilon$, be given. Then we can write

$X = a^m b$, $-1 = a^n b$ when $m+n$. Take $Z = b^m$.

Thus $XZ = a^m b^m \in L_3$, and $YZ = a^n b^m \notin L_3$. 

...
**Defn:** A string \( w \in \{a, b\}^* \) has a "balancing b" if we can write \( w = uvbx \) such that \( |u| = |v| \), \( a \bar{b}ab \).

The same proof for \( L_3 \) works to show that both versions of this definition produce non-regular languages.

- For any \( K \geq 1 \), define \( L_K = \{ x \in \{0, 1\}^* : \) the \( K \)-th bit from the right is a 1 \}. 

\( L_3 \) is the language of this NFA:

![NFA Diagram]

There is nondeterminism on \( 1 \) at the start state. Given a string such as \( x = 10110 \), the machine has to guess which 1 (if any) is 3rd from the end — by taking the "leap of faith" on an access to state 1 on it. If it leaps too soon, it will crash in state 3 rather than be there when the string stops. If it doesn't leap or leaps too late, it will end in state 0, 1, or 2 — no good either. Anyway, \( L(N_3) = L_3 \), and for any \( K \), one can similarly build \( N_K \) with \( K+1 \) states so that \( L(N_K) = L_K \).

How many states does a DFA \( M_K \) need so that \( L(M_K) = L_K \)?

**Proposition:** \( S = \{0, 1\}^K \) is a PD set of size \( 2^K \) for \( L_K \).

**Proof:** Let any \( x \in S \). \( x \neq y \), be given. Then there is a bit place \( i \), \( 0 \leq i \leq K-1 \), in which they differ. Without loss of generality suppose \( x \) is the string with a 0 here: \( x = 0y \).

Take \( z = 0^i \). Then \( xz \) has a 0 in place \( K \)-th from end, so \( xz \notin L_K \), but \( yz \) has a 1 there so \( yz \in L_K \). So \( L_K(xz) \neq L_K(yz) \), so \( S \) is PD. Thus \( M_K \) needs (at least) \( 2^K \) states.