Myhill Nerode Examples (Wed Sept. 23 after console crashed)

$L_0 = \{a^n b^n : n \geq 0\}$. Take $S = \{a^n : n \geq 0\} = a^*$
Clearly S is infinite.

Let any $x, y \in S$, $x \neq y$ be given. Then we can write

$x = a^m$, $y = a^n$, where $m \neq n$ [If we care, we may suppose that $m < n$ w.l.o.g.]

Take $z = b^m$. Then $xz = a^m b^m \in L_0$
But $yz = a^n b^m \notin L_0$ because $n \neq m$.

Hence S is PD for L_0, and since S is infinite, L_0 is not regular by MNT. $\boxtimes$

$L_1 = \{\$^g D^h : g \geq h\}$. Take $S = \$^*$, clearly infinite.

Let any $x, y \in S$ be given. Then $x = \$^m$, $y = \$^n$ where w.l.o.g. $m < n$.
Take $z = D^n$? Then $xz = \$^m D^n \notin L_1$ Here we do need to care

So $L_1(xz) \neq L_1(yz)$ and $yz = \$^n D^n \in L_1$

and since $x, y \in S$ are arbitrary (w.l.o.g.) as S is infinite, L_1 is not regular

L_2 = the "arbitrary spears" game in general, with empty rooms O and any order of $\$$s and Ds allowed.

$L_3 = \{x \in \{a,b\}^* : x$ is a palindrome, ie. $x = x^R\}$. Examples
abba ϵ "a" "b" baba not a palindrome.

Take $S = a^*b$. Let any $x, y \in S$, $x \neq y$, be given. Then we can write
$x = a^m b$, $y = a^n b$ where $m \neq n$. Take $z = a^m$. Then $xz = a^m b a^m \in L_3$
but $yz = a^n b a^m \notin L_3$.

Defn: A string $w \in \{a,b\}^*$ has a "balancing b" if we can write $w = ubv$ such that ~~$\#a(u) = \#a(v)$~~. $|u| = |v|$.

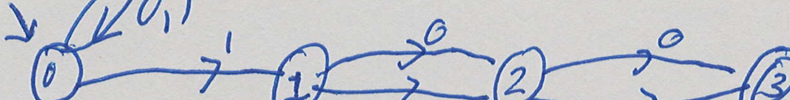
abbab

★ The same proof for L_3 works to show that both versions of this definition produce non-regular languages.

[This example was intended but will be covered Friday.]

For any $k \geq 1$, define $L_k = \{x \in \{0,1\}^* :$ the kth bit from the right is a $1\}$.

L_3 is the language of this NFA: $N_3 =$



There is nondeterminism on '1' at the start state. Given a string such as $x = 10110$, the machine has to guess which 1 (if any) is 3rd from the end — by taking the "leap of faith" arc across to state 1 on it. If it leaps too soon, it will crash in state 3 rather than be there when the string stops. If it doesn't leap or leaps too late, it will end in state 0, 1, or 2 — no good either. Anyway, $L(N_3) = L_3$, and for any k, one can similarly build N_k with $k+1$ states so that $L(N_k) = L_k$.

$\boxed{\text{How many states does a DFA } M_k \text{ need so that } L(M_k) = L_k\text{?}}$

Proposition: $S = \{0,1\}^k$ is a PD set of size $\boxed{2^k}$ for L_k.

Proof: Let any $x, y \in S$, $x \neq y$, be given. Then there is a bit place i, $0 \leq i \leq k-1$, in which they differ. Without loss of generality suppose x is the string with a 0 there: $x_i = 0$, $y_i = 1$. Take $z = 0^i$. Then xz has a 0 in place kth from end, so $xz \notin L_k$, but yz has a '1' there so $yz \in L_k$. So $L_k(xz) \neq L_k(yz)$, so S is PD. ∎ Thus M_k needs (at least) 2^k states.