CSE491/596, Fall 2020 Problem Set 1 Due Mon. Sept. 21*

Lectures and Reading. Monday's lecture will finish the proof of Kleene's Theorem. For that, please read section 3 of Debray's notes in-tandem with the GNFA coverage in Ogihara's notes on Regular Expressions on the course webpage. Then for Wednesday, beginning coverage of the Myhill-Nerode Theorem, *skim* section 4 and focus on section 5 of Debray's notes. Also read the Myhill-Nerode handout on the course webpage. You can read ahead to section 6 of Debray's notes if you wish, but it will be "covered" only *after* Turing Machines are formally introduced in week 4.

(1) Suppose we change the rules of the "spears and dragons game" to read as follows:

- Each character in the input string—figuratively, each char is a "room" in a linear "dungeon"—is either \$ for "spear," D for "dragon," or ℓ for "lamp."
- The Player P may hold a maximum of **2** spears at any one time.
- Upon entering a room with a spear, P may pick it up unless already carrying two spears.
- Upon entering a room with a dragon, if P has no spear, P is dead. Else, P uses one spear to kill the dragon, and is then carrying one fewer spear.
- Upon entering a room with a lamp, if P has killed two dragons since the last time P picked up a spear (it follows that P currently has no spear), then P may rub the lamp and the genie in the lamp will give P one spear. Otherwise, the lamp has no effect.

Design a deterministic finite automaton M that simulates this game. In particular, $L(M)$ should equal the language of strings representing "dungeons" that P survives. The final state of M should also tell the number of spears that P has when exiting the dungeon. (If you sense an ambiguity in the rules, you may ask on *Piazza*, and/or you may describe the perceived ambiguity and detail the interpretation you took to resolve it. 18 pts.)

(2) Convert the following NFA N with ϵ-transitions into an equivalent DFA. The code for N has $Q = \{1, 2, 3, 4\}$, $\Sigma = \{a, b\}$,

$$\delta = \{ (1, \epsilon, 2), (1, a, 3), (2, a, 2), (2, b, 4), (3, b, 2), (3, b, 4), (4, a, 4), (4, b, 1) \},$$

$s = 1$, and $F = \{2\}$. (18 pts.)

(3) Calculate a regular expression over $\Sigma = \{a, b\}$ for the language of strings that are **not** accepted by the following NFA: $Q = \{s, q, f\}$, $F = \{f\}$, and

$$\delta = \{(s, a, q), (s, b, f), (q, b, s), (q, a, f), (f, a, s), (f, \epsilon, q)\}.$$

(Note that if the last instruction were on b not ϵ it would be a DFA.) You must use a strategy based on theorems in lectures and posted notes, not just inspection (that is, "hacking"). (24 pts., for 60 total on the written part of the set)

(4) * *There will be a further problem or sequence of problems for presentation or further discussion; I have not yet settled how to handle them; due times will vary.*