Review Session Notes on the Myhill-Nerode Theorem

These are best viewed with the Review Session recording https://ub.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=88581f77-62ca-4388-957b-b09700197f0d

$L_2 = \{x0y : |x| = |y|\}$ Re-cast this to say:

$L_2 = \{w : w \text{ can be broken as } w =: x0y \text{ such that } |x| = |y|\}$

$L_2 = \{w : w \text{ can be broken as } w =: u0v \text{ such that } |u| = |v|\}$.

Take $S = 0^*$. Clearly infinite. Let any $x, y \in S$ $(x \neq y)$ be given. Then we can represent them wlog. as $x = 0^i$, $y = 0^j$ where $i < j$. MisTake $z = 00^i$. Then $xz = 0^i 00^i$ is in L_2 but (this is the trap): $yz = 0^j 00^i$ which is not in L_2 because $j \neq i$. Refutation: The case $i = 3, j = 5$ is a possible one for our general choice. Then $yz = 00000 \cdot 0 \cdot 000$, however, this string also can be broken as $0000 \cdot 0 \cdot 0000$ and so it does belong to L_2 after all. Correct: take $z = 01^j$ using the larger number of $1s$. Now it is $yz = 0^j 01^j$ that belongs to L_2, whereas $xz = 0^i 01^j$ cannot belong because even if $i + j$ is even, there are too many 1s to break it with a 0 in the middle. (E.g. with the same $i = 3, j = 5$ values, $xz = 000011111$.)

Common mistake on both, but especially saw it on L_3: the "Too Many Stars" problem.
$L_3 = \left\{uv : u \oplus v = 1^{|u|}\right\}$. In view of the basic idea that $00000 \oplus 11111 = 11111,$ the temptation: take $S = 0^*1^*$. Problem: A general choice of strings x, y in this S has the form:

$x = 0^p 1^q$
$y = 0^r 1^s$, where all you get from $x \neq y$ is that $p \neq r$ OR $q \neq s$.

Taking $S = \left\{0^n 1^n : n \geq 0\right\}$ is OK from the degrees of freedom point of view: a general choice is
$x = 0^p 1^p$
$y = 0^r 1^r$ where $p \neq r$ (and wlog. you can say $p < r$).
Then this works with $z = 1^p 0^p$ again, without needing the "wlog.", but is more complicated than the key answer taking simply $S = 0^*$.