

Open book, open notes, closed neighbors, 48 minutes. The exam totals 80 pts., subdivided as shown. Do em all three problems in the exam book provided—there is no “choice” option. em Show your work—this may help for partial credit.

(1) (5 × 4 = 20 pts. total) True/False.

**Note for Fall 2020:** These 20 points would be absent from your exam. Also parts of problem 2, especially (d), are more advanced than I’ve covered in 491/596, but they are good for review.

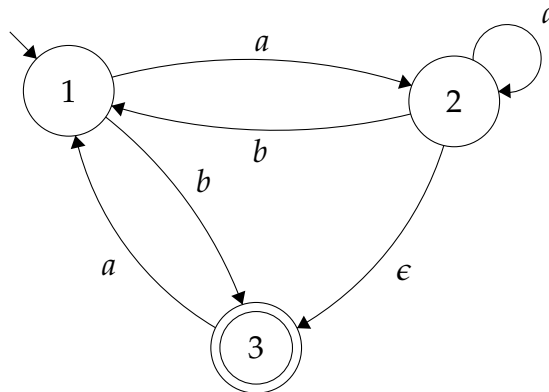
Please write out the words *true* and *false* in full. Brief justifications are not necessary but may help for partial credit. Given sets  $A$  and  $B$ , the difference  $A \setminus B$  is the same as  $A \cap \bar{B}$ . the right it is. em Please write in exam books only.

- (a) If  $A$  and  $B$  are regular, then  $A \setminus B$  is always regular.
- (b) If  $A$  and  $B$  are decidable, then  $A \setminus B$  is always decidable.
- (c) If  $A$  and  $B$  are computably enumerable, then  $A \setminus B$  is always computably enumerable.
- (d) If  $A$  is regular, then  $A^*$  is decidable in linear time by a single-tape Turing machine.
- (e) Every non-regular language is decidable.

(2) 18 = 12 = 30 pts.

Consider the following nondeterministic finite automaton  $N = (Q, \Sigma, \delta, s, F)$  where  $Q = \{1, 2, 3\}$ ,  $\Sigma = \{0, 1\}$ ,  $s = 1$ ,  $F = \{3\}$ , and the instructions in  $\delta$  are:

$$\{(1, a, 2), (1, b, 3), (2, a, 2), (2, b, 1), (2, \epsilon, 3), (3, a, 1)\}.$$



Convert  $N$  into a DFA  $M$  such that  $L(M) = L(N)$  (18 pts.). Also answer the following questions (3 pts. each).

- (a) Is there a string  $u$  such that for each of its states  $q$ ,  $N$  can process  $u$  from 1 to  $q$ ? Give a shortest such string if so.
- (b) Is there a string  $v$  that  $N$  cannot process starting from state 1 at all? Again give a shortest such string if so.
- (c) Is there a string  $w$  such that for all  $y \in \Sigma^*$ ,  $wy \in L(N)$ ? Again give a shortest  $w$  if so.
- (d) Does  $L(N)$  include  $b(aaabb)^*$ ? Briefly justify from your  $M$ .

**(3) (8 + 4 + 18 = 30 pts.)**

Define  $L$  to be the language of strings  $x$  such that  $|x|$  is even and the second half of  $x$  contains at least one '1.' For instance 010100 is in  $L$  but 01010000 is not, and 0100001 is not because its length is odd.

- (a) Which of the following strings belong to  $L$ ? Say yes/no for each.

(i)  $\epsilon$       (ii) 1      (iii) 01      (iv) 010.

- (b) Is  $L \cdot L \subseteq L$ ? Justify your answer briefly.
- (c) Prove via the Myhill-Nerode Theorem that  $L$  is nonregular.

END OF EXAM.