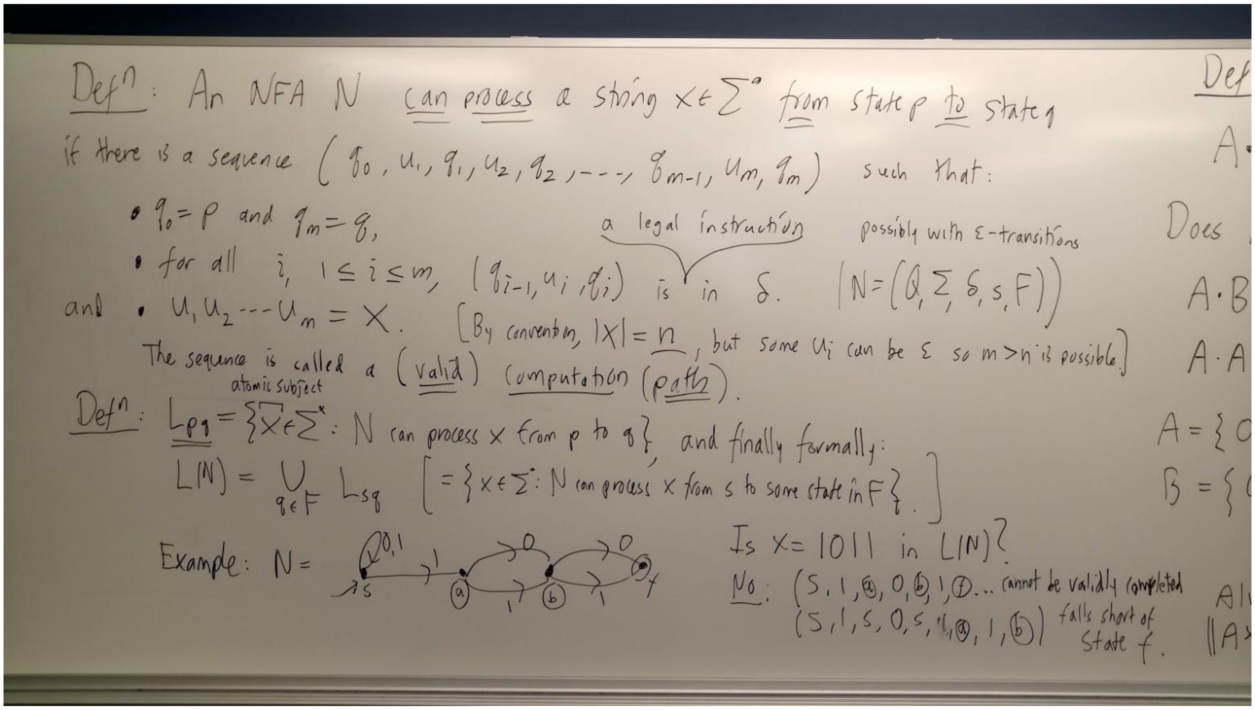
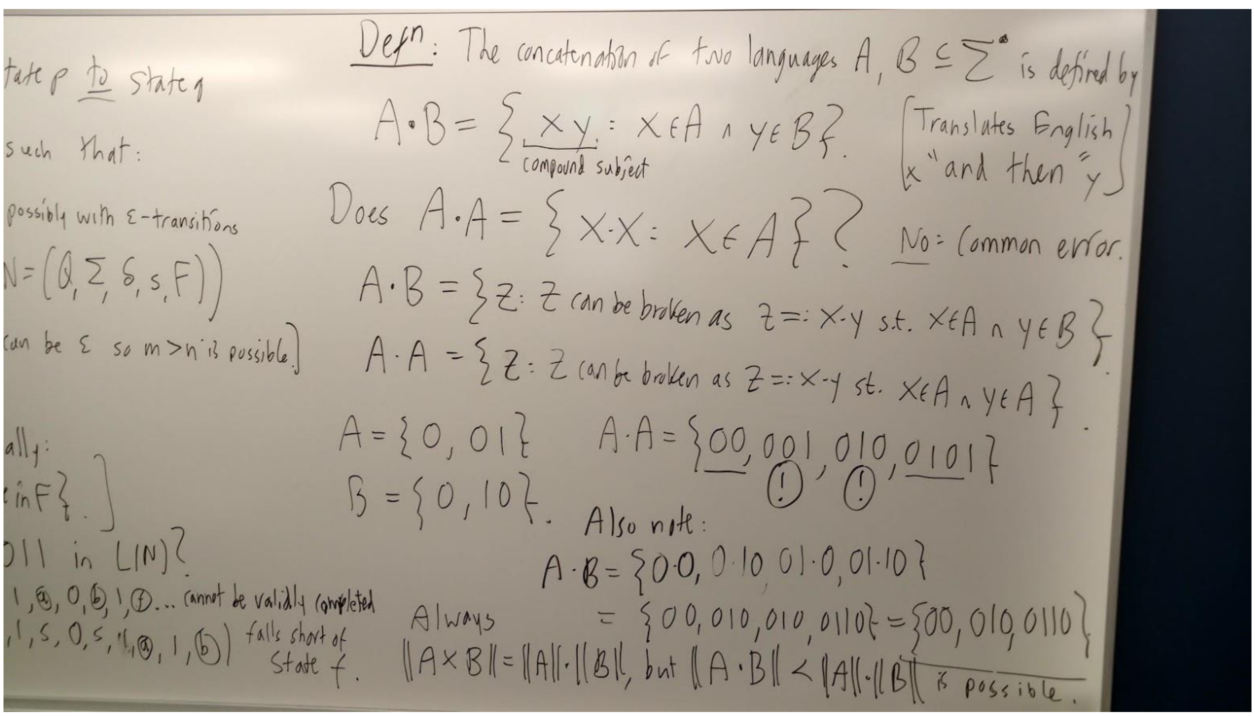
CSE596 Lecture Sept. 4, 2019





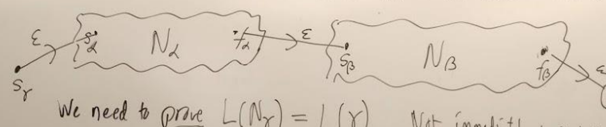
Picking up in the inductive cases of the proof, given regexps α, β and equivalent NFAs N_α, N_β.

• $\gamma = \alpha \bullet \beta$ is a regexp,

$L(\gamma) =_{def} L(\alpha) \cdot L(\beta)$,

and given N_α, N_β, build $N_\gamma =$

"Series Circuit": Could identify $s_\gamma \equiv s_\alpha$, $f_\gamma \equiv f_\beta$, but the middle ε-arc can be necessary



We need to prove $L(N_\gamma) = L(\gamma)$. Not immediately obvious! Note the flow.

$L(N_\gamma) = \{ z \in \Sigma^* : N_\gamma$ can process z from s_γ to $f_\gamma \}$.

By the particular series form of N_γ, $L(N_\gamma) = \{ z \in \Sigma^* :$ z has a prefix x that gets processed from s_γ via s_α to f_α, then the bridge ε to s_β, then a suffix y processed from s_β to f_β before the ε to $f_\gamma \}$.

Induction hypothesis about correctness of the NFAs is used here ↘

$= \{ z \in \Sigma^* : z$ can be broken as $z =: x \cdot y$ st. $x \in L(N_\alpha) \wedge y \in L(N_\beta) \}$

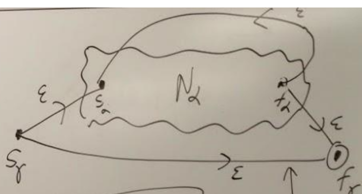
$= L(N_\alpha) \cdot L(N_\beta)$

$\overset{!}{=} L(\alpha) \cdot L(\beta) =_{def} L(\gamma)$. $\therefore L(N_\gamma) = L(\gamma)$, as needed to propagate the induction hypothesis.

• γ = / L(γ) / A* =def / A·B / A·A / A = {0 / B = { / Alw / ||A×

...quivalent NFAs N_α, N_β.
f_β, but the middle ε-arc can be necessary
N_β f_β ε f_γ
Not immediately obvious! Note the flow.
from s_γ to $f_\gamma \}$.
...hat gets processed from s_γ via s_α
ε to s_β, then a suffix y processed
before the ε to $f_\gamma \}$.
$z =: x \cdot y$ st. $x \in L(N_\alpha) \wedge y \in L(N_\beta) \}$
$L(\gamma)$. $\therefore L(N_\gamma) = L(\gamma)$,
as needed to propagate the induction hypothesis.

• $\gamma = \alpha^*$ is a regexp. N_γ



$L(\gamma) = L(\alpha)^*$ defined via

$A^* =_{def} A^0 \cup A^1 \cup A^2 \cup A^3 \cup \cdots$

$= \{\varepsilon\} \cup A \cup A \cdot A \cup A \cdot A \cdot A$

Feedback Circuit (with bypass). ⊠

$A \cdot B = \{ z : z$ can be broken as $z =: x \cdot y$ s.t. $x \in A \wedge y \in B \}$.

$A \cdot A = \{ z : z$ can be broken as $z =: x \cdot y$ st. $x \in A \wedge y \in A \}$.

$A = \{0, 01\}$ $A \cdot A = \{00, 001, 010, 0101\}$ (!) (!)

$B = \{0, 10\}$. Also note:

$A \cdot B = \{0 \cdot 0, 0 \cdot 10, 01 \cdot 0, 01 \cdot 10\}$

$= \{00, 010, 010, 0110\} = \{00, 010, 0110\}$

Always $\|A \times B\| = \|A\| \cdot \|B\|$, but $\|A \cdot B\| < \|A\| \cdot \|B\|$ is possible.