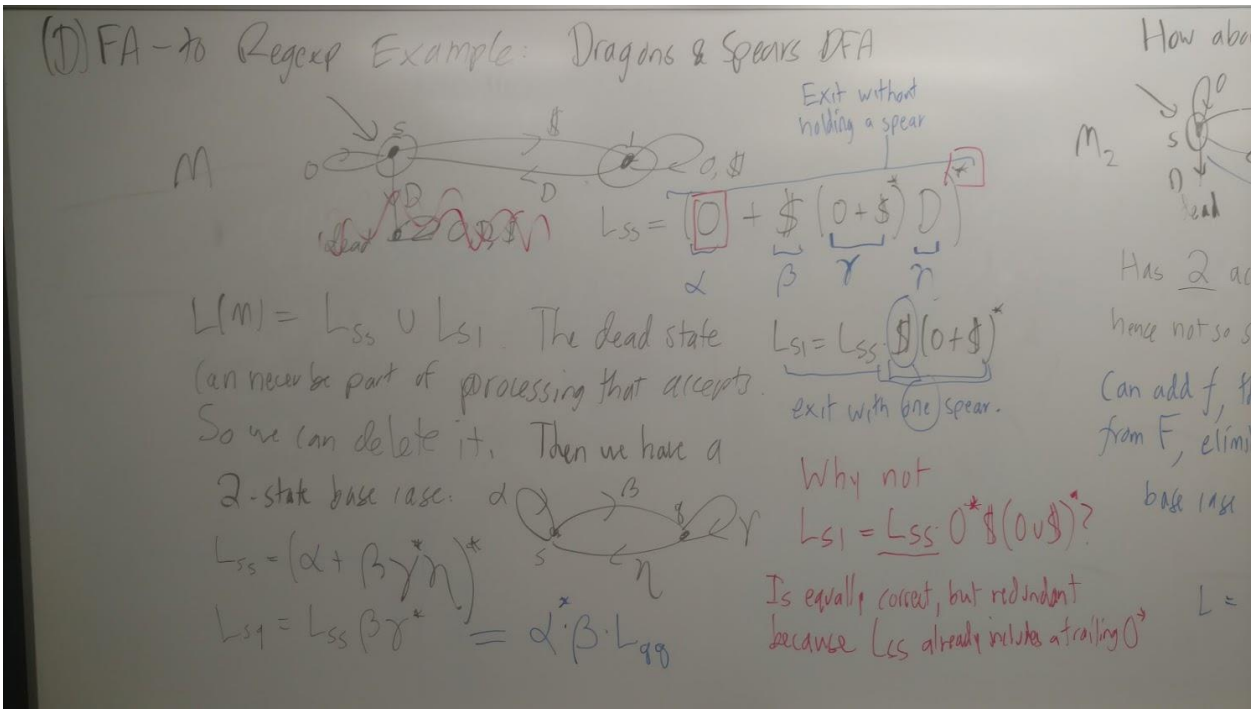
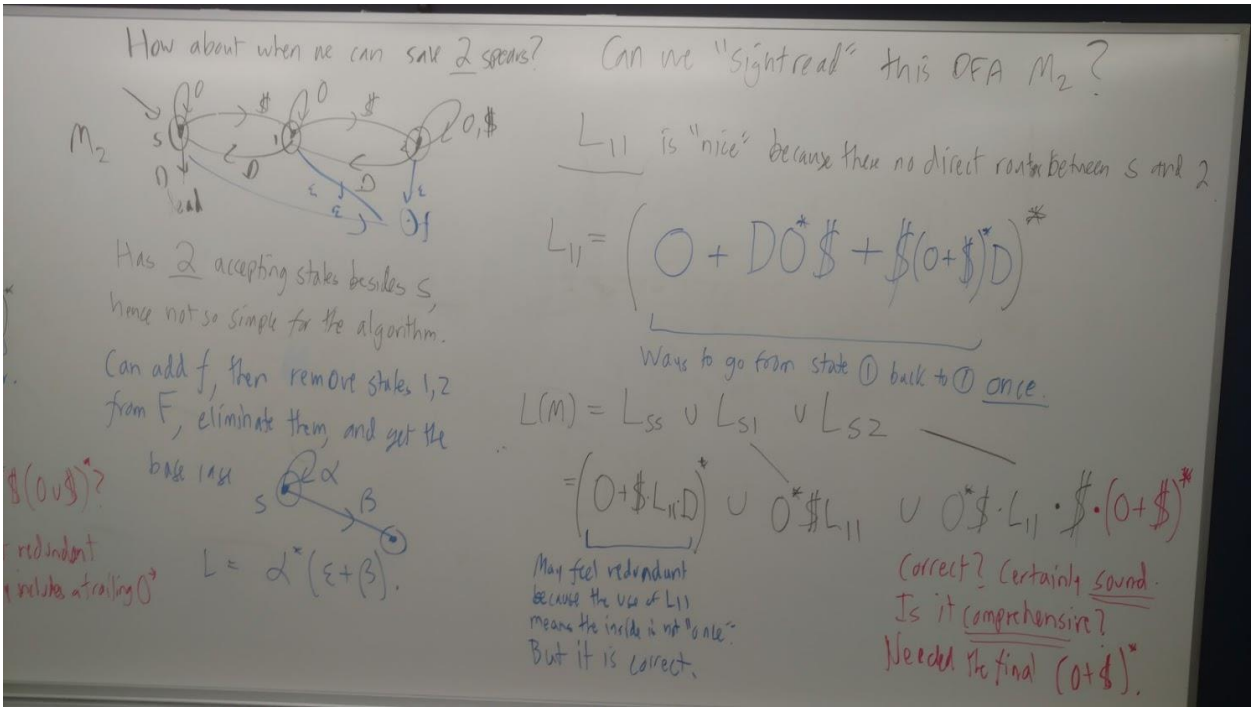
CSE596 Lecture Wed. Sept. 11, 2019





Definition: Given any language L (not necessarily regular), two strings $x, y \in \Sigma^*$ are *distinctible for L*, written $x \not\sim_L y$, if there is a string $z \in \Sigma^*$ s.t. $L(xz) \neq L(yz)$.

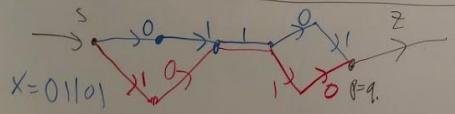
Negation is $(\forall z \in \Sigma^*)\ L(xz) = L(yz)$, then write $x \sim_L y$.

Identifying $L(\cdot)$ with the Boolean valued fn $\begin{cases} 1 & \text{if } x \in L \\ 0 & \text{otherwise} \end{cases}$

Nicer than writing $xz \in L$ XOR $yz \in L$

i.e. $(xz \in L \cap yz \notin L)$ or $(xz \notin L \cap yz \in L)$

Key Point: If $x \not\sim_L y$, then any DFA M such that $L(M) = L$ must process x, y from s to different states p, q.



$x = 01101$

$y = 10110$

Upon starting any suffix z, the processing would stay converged, so xz and yz would end at the same state r. But r cannot be both acc and rej.

Then M could not be correct for L.

How about when n

E.g. $w = \varepsilon$

Now suppose we hv

w, x, y st. $wx \not\sim_L$

Then we need 3 d

to process them to

Defn: A set $S \subseteq \Sigma^*$ is

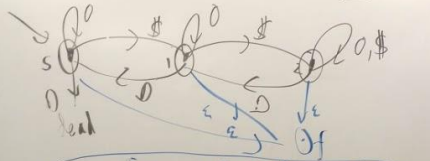
for L if for all distinct

$x \not\sim_L y$.

Key Lemma: If $\|S\| = k$,

s.t. $L(M) = L$ must have

How about when we can save 2 spaces?



E.g. $w = \varepsilon$, $x = \$$, $y = \$\$$.

Now suppose we have 3 strings w, x, y st. $wx \not\sim_L x$, $wx \not\sim_L y$, $x \not\sim_L y$.

Then we need 3 different states to process them to.

Defn: A set $S \subseteq \Sigma^*$ is *pairwise distinctive* / *PD* for L if for all distinct pairs $x, y \in S$, $x \not\sim_L y$.

Key Lemma: If $\|S\| = k$, then any DFA M s.t. $L(M) = L$ must have at least k states.

Then M could not be correct for L.

John T 1997, UB Anil still alive, at Cornell

Myhill-Nerode Theorem

not rec. $S \subseteq L$.

(a) If a language L has an infinite PD set S, then L is not regular.

(b) Moreover, every nonregular lang. has an infinite PD set.

Proof for (a): Read the "Key Lemma" with $k = \infty$.

Every M would need ∞-many states, but then M would not be a *finite* automaton. So L has no DFA, so L is not regular.

Note also that $\{\varepsilon, \$, \$\$, D\}$ is a PD set of size 4, so M_2 is optimal.