CSE596 Lecture Fri 9/21 Fall 201

Nonregularity via MNT Examples.

$A = \{0^n 1^n : n \geq 1\}$ $A = B \cap 0^+ 1^+$

$B = \{x \in \{0,1\}^* : \#0(x) = \#1(x), x \neq \epsilon\}$.

Take $S = 0^+$. Clearly S is infinite. Let any $x, y \in S$, $x \neq y$, be given. Then we can write $x = 0^m$, $y = 0^n$ where $m \neq n$, $m, n \geq 1$. Take $z = 1^m$. Then $xz = 0^m 1^m \in A$ (and $\in B$), but $yz = \underline{0^n 1^m} \notin A$ since $m \neq n$ (and $\notin B$ di...

Thus $A(xz) \neq A(yz)$, so S is an infinite PD set for A, and $B(xz) \neq B(yz)$, " " " for B, so $A, B \notin$ REG.

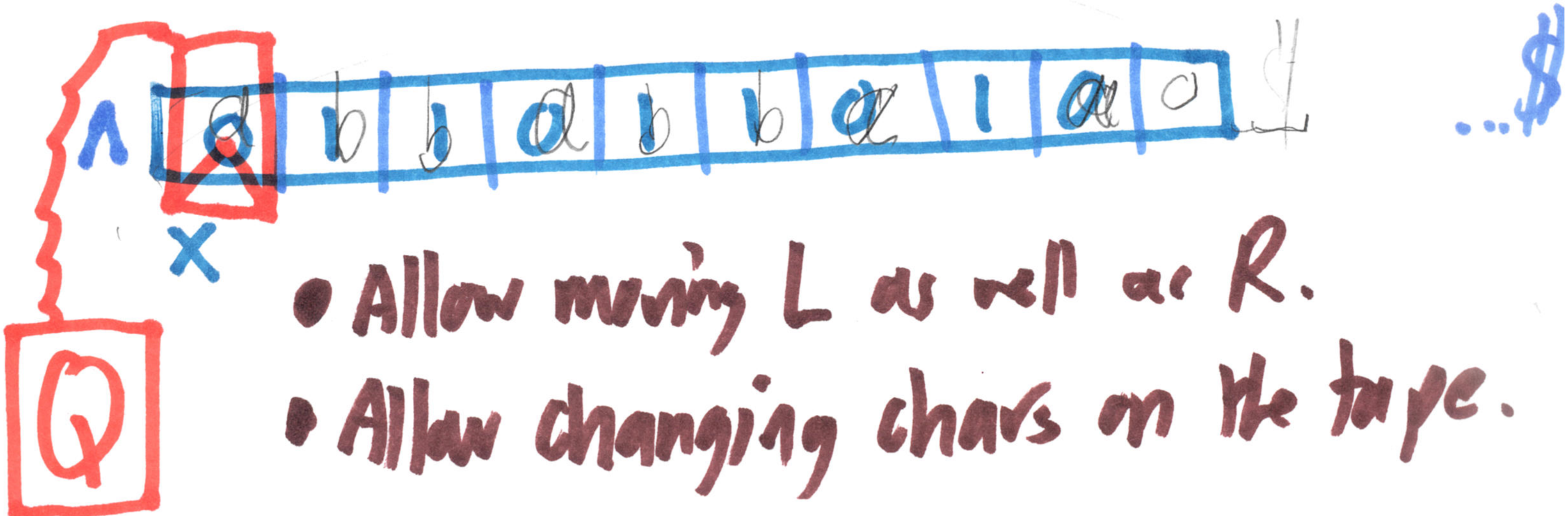
The same proof also goes for the complements $\tilde{A}$ and $\tilde{B}$: $\tilde{A}(xz) \neq \tilde{A}(yz) \iff A(xz) \neq A(yz)$.

But the Pumping Lemma is very tricky on B in particular.

$A' = \{0^{\cancel{m}\,k} 1^{\cancel{n}\,\ell} : \cancel{m} \geq \cancel{n}\ \ k \geq \ell\}$ OK if we say "wlog $n < m$" in the proof.

So DFAs cannot recognize languages like A or B or "Dragon$L where you can save any # of spears".

How can we liberalize DFAs to do so?



- Allow moving L as well as R.
- Allow changing chars on the tape.

Doing just one does not increase power beyond DFAs

Doing both defines a (deterministic) Turing Machine.

Alan Turing, 1936

$\Gamma = \{0,1\} \cup \{a, b, \sqcup\}$ ← blank. also have $, ^.

Defn: A TM is $M = (Q, \Sigma, \underline{\Gamma}, \delta, \underline{\sqcup}, s, F)$

blank "stay"

where Γ contains Σ and the blank $\sqcup$ and

$$\delta \subseteq (Q \times \Gamma) \times (\Gamma \times \{L, R, S\} \times Q)$$

Instruction $(p, c \,/\, d, D, q)$

$d = c$ allowed.

If M is in state p scanning char c, it WILL change c to d, move its head L, R, or S and goto state q.

M is a DTM if δ defines a function from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R, S\}$.

Def^n: The DTM is in "nice form" if

$F = \{q_{acc}\}$ for one $q_{acc} \in Q$

there is another state <u>q_{rej}</u> with no out-arcs

$\delta : Q \setminus \{q_{acc}, q_{rej}\} \times \Gamma \to \Gamma \times \{L, R, S\} \times Q$

Added: Preview for Monday:

<u>Def^n</u>: A k-tape TM has the same $M = (Q, \Sigma, \Gamma, \delta, \sqcup, s, F)$

but $\delta \subseteq (Q \times \Gamma^k) \times (\Gamma^k \times \{L, R, S\}^k \times Q)$.

I like to write multi-tape tuples with chars vertical to imitate the tapes:

Typical tuple: $\left(p, \begin{matrix} c_1 \\ \vdots \\ c_k \end{matrix} / \begin{matrix} d_1 \\ \vdots \\ d_k \end{matrix}, \begin{matrix} D_1 \\ \vdots \\ D_k \end{matrix}, q\right)$

Most sources put the destination Q part in the middle but the end is best graphically.

Graphically: $k = 2$

$$p \xrightarrow{\left(\begin{matrix} c_1 \\ c_2 \end{matrix} / \begin{matrix} d_1, R \\ d_2, L \end{matrix}\right)} q$$

$p = q$ is a self-loop.

Nice form with $F = \{q_{acc}\}$, $\delta : (Q \setminus \{q_{acc}, q_{rej}\}) \times \dots$ is as above. Computations are weirder to define but the $L(M) =_{def} \{x \in \Sigma^* : M(x)$ has a computation that halts at q_{acc}