

Theorem: For every k -tape TM $M_k = (Q, \Sigma, \Gamma, \delta, q_0, s, \{q_i, r_i, \dots\})$ we can build a 1-tape TM $M_1 = (Q', \Sigma', \Gamma', \delta', q_0', s', \{q_i', r_i', \dots\})$ st. $L(M_1) = L(M_k)$ and M_1 simulates t steps of M_k in $O(t^2)$ steps of its own.

Proof: Idea: Put k "trackers" on one tape, including "dotted chars" to mark where the k tape heads of M_k are.

Take $\Gamma' = \Gamma \cup (\Gamma \cup \{ \cdot \})^k \cup \{ \cdot \}$. $\Gamma' = \{c_i : c_i \in \Gamma\}$ Includes \square , which $\neq \square$. Moreover, Γ' has $\square^k = \begin{pmatrix} \square \\ \square \\ \vdots \\ \square \end{pmatrix}$.

1. $\square x_1 x_2 \dots x_n \square \dots$
 2. $\square \square \dots$
 ...
 i. $\square \square$
 ...
 k. $\square \square$
 $\square \square \dots (x)$

$(n+2)$ steps later

IF M_k really wants to write an a on tape 1 after x_n , M_1 writes $\begin{pmatrix} a \\ \square \\ \vdots \\ \square \end{pmatrix}$ instead and changes the rest \square to \square .

Every REW cycle uses at most $s + 3s$ steps = $O(s)$ steps, where s is the current distance from Λ to $\$$. Since $s \leq n+t$ when M_k has run for t steps, the total time $t \cdot O(s) \leq t^2 + tn = O(t^2)$.

a 1-tape TM M_1 simulates t steps of M_k in $O(t^2)$ steps of its own.

\square , which $\neq \square$. Moreover, Γ' has $\square^k = \begin{pmatrix} \square \\ \square \\ \vdots \\ \square \end{pmatrix}$ and $\begin{pmatrix} \square \\ \square \\ \vdots \\ \square \end{pmatrix}$.

IF M_k really wants to write an a on tape 1 after x_n , M_1 writes $\begin{pmatrix} a \\ \square \\ \vdots \\ \square \end{pmatrix}$ instead and changes the rest \square to \square .

Procedure by M_1 :

- Initialize tape to $\Lambda \begin{pmatrix} \square \\ \square \\ \vdots \\ \square \end{pmatrix} x_1 \dots x_n \square$ before starting to simulate instructions of $M_k(x)$. It is in the ID with the start state s of M_k .
- Begin a Read-Evaluate-Write (REW) cycle that always begins at a state q of M_k and ends at a state r of M_k . At the begin \square of M_1 is to right of Λ and all heads of M_k are between that and the $\$$.
 - Read while sweeping L to R the k -tuples $c_1 \dots c_k$ of chars scanned by M_k .
 - Execute the corresp instruction $(q, \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix}, \begin{pmatrix} d_1 \\ \vdots \\ d_k \end{pmatrix}, r)$ in a R to L sweep from $\$$ back to Λ . Write changes to the tape by M_k using \square s as needed.

$t \cdot s + 3s$ steps = $O(s)$ steps, where s is the current distance from Λ to $\$$. Since $s \leq n+t$ when M_k has run for t steps, the total time $t \cdot O(s) \leq t^2 + tn = O(t^2)$.

Also some discussion of Assignment 1:

$L_3 = \{x \mid x \text{ is a (possible multiple of 3)}\}$
 without positive, $\epsilon \in L_3$
 Then L'_3 becomes Σ^*
 and the problem is trivial.

states t steps of M_K
 steps of its run.

more blocks in that column being scanned, or none.

reverse, Γ^1 has $\begin{pmatrix} D \\ D \\ \vdots \\ D \end{pmatrix}$ and $\begin{pmatrix} D \\ D \\ \vdots \\ D \end{pmatrix}$

$L'_3 = (0+1)^* L_3 (0+1)^*$

SS aftermost (*)
 run U or + B overmost.

$L_{S2} \cup L_{S3}$
 $= L_{S2} \cdot 1^*$

still has a + or U
 goto 3

To do $(0+1)^* L(N) (0+1)^*$

OK provided you make the fin the diagram
 not have any arcs to a non-acc state