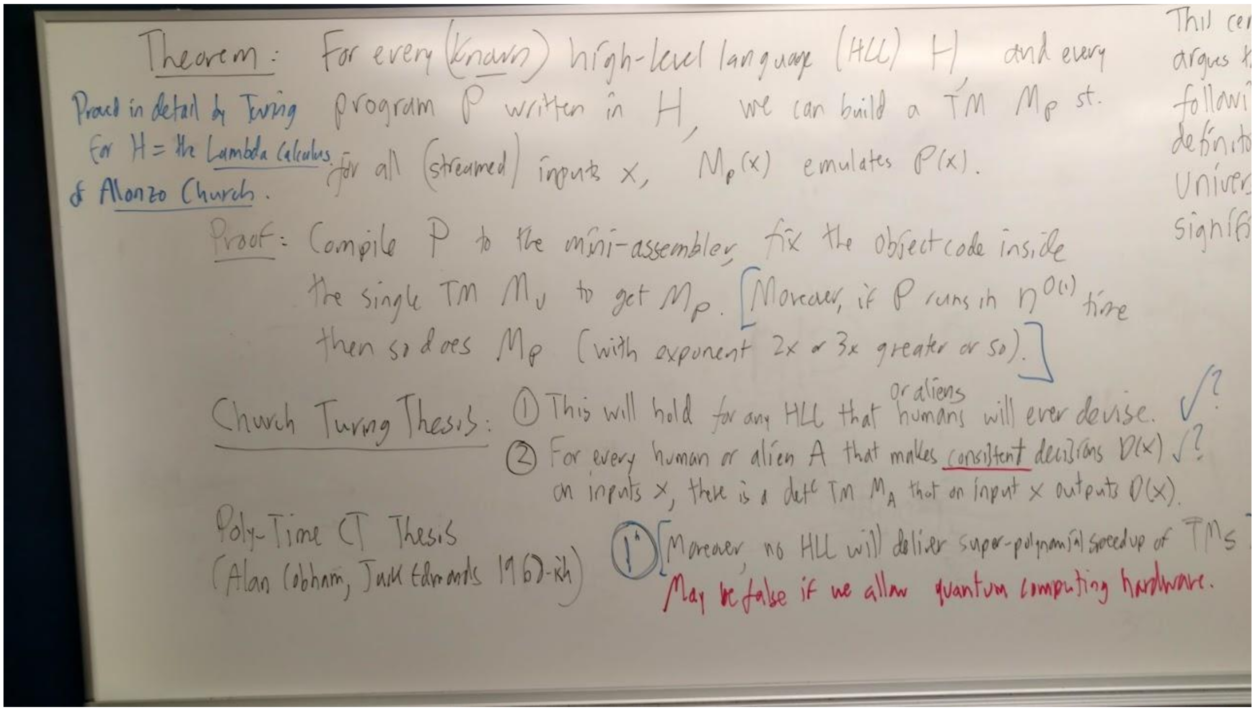
CSE596 Lecture Wed. Sept. 25, 2019



Theorem: For every (known) high-level language (HLL) H, and every program P written in H, we can build a TM M_P s.t. for all (streamed) inputs x, $M_P(x)$ emulates $P(x)$.

Proved in detail by Turing for H = the Lambda Calculus of Alonzo Church.

Proof: Compile P to the mini-assembler, fix the object code inside the single TM M_U to get M_P. [Moreover, if P runs in $n^{O(1)}$ time then so does M_P (with exponent 2x or 3x greater or so).]

Church Turing Thesis: ① This will hold for any HLL that humans or aliens will ever devise. √?

② For every human or alien A that makes consistent decisions $D(x)$ √? on inputs x, there is a det TM M_A that on input x outputs $D(x)$.

Poly-Time CT Thesis (Alan Cobham, Jack Edmonds 1962-ish) ①' [Moreover, no HLL will deliver super-polynomial speedup of TMs]

May be false if we allow quantum computing hardware.

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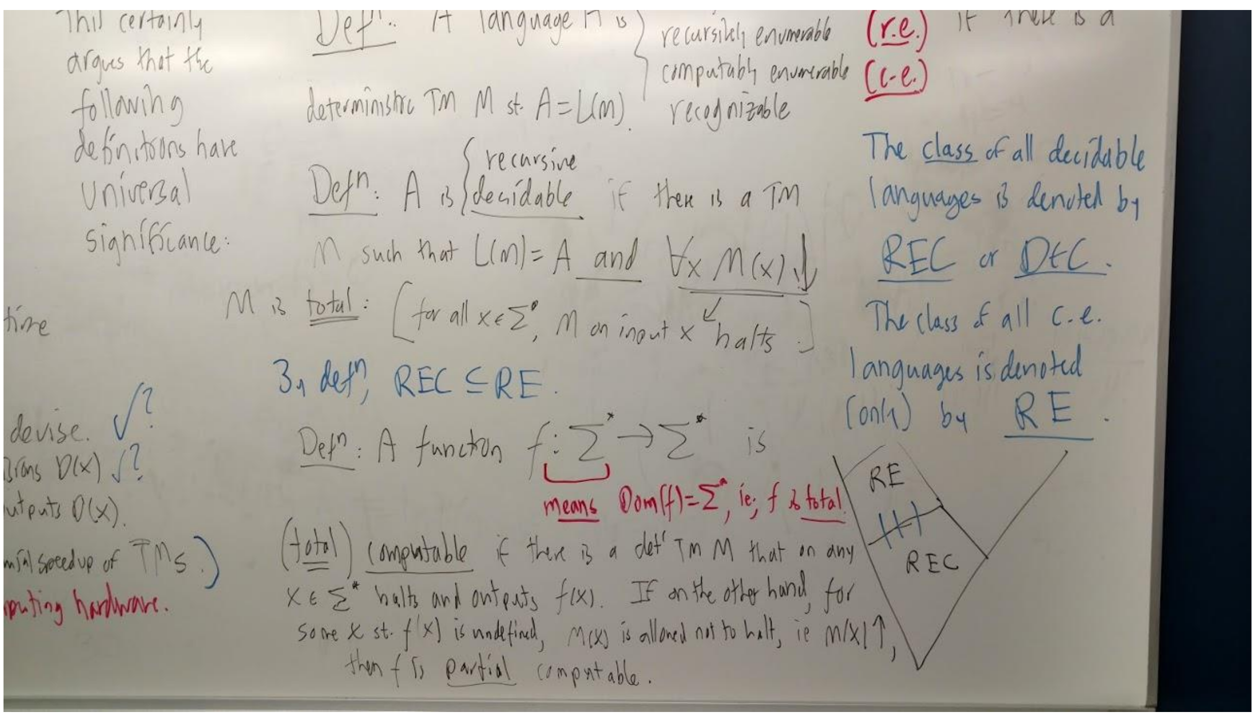

This certainly argues that the following definitions have universal significance:

Defn: A language A is recursively enumerable (r.e.) / computably enumerable (c.e.) / recognizable if there is a deterministic TM M s.t. $A = L(M)$.

Defn: A is recursive / decidable if there is a TM M such that $L(M) = A$ and $\forall x\ M(x)\downarrow$.

M is total: [for all $x \in \Sigma^*$, M on input x halts]

By defn, $REC \subseteq RE$.

Defn: A function $f: \underbrace{\Sigma^*}_{\text{means } Dom(f)=\Sigma^*\text{, ie, } f \text{ is total}} \to \Sigma^*$ is (total) computable if there is a det TM M that on any $x \in \Sigma^*$ halts and outputs $f(x)$. If on the other hand, for some x s.t. $f(x)$ is undefined, $M(x)$ is allowed not to halt, ie $M(x)\uparrow$, then f is partial computable.

The class of all decidable languages is denoted by REC or DEC.

The class of all c.e. languages is denoted (only) by RE.

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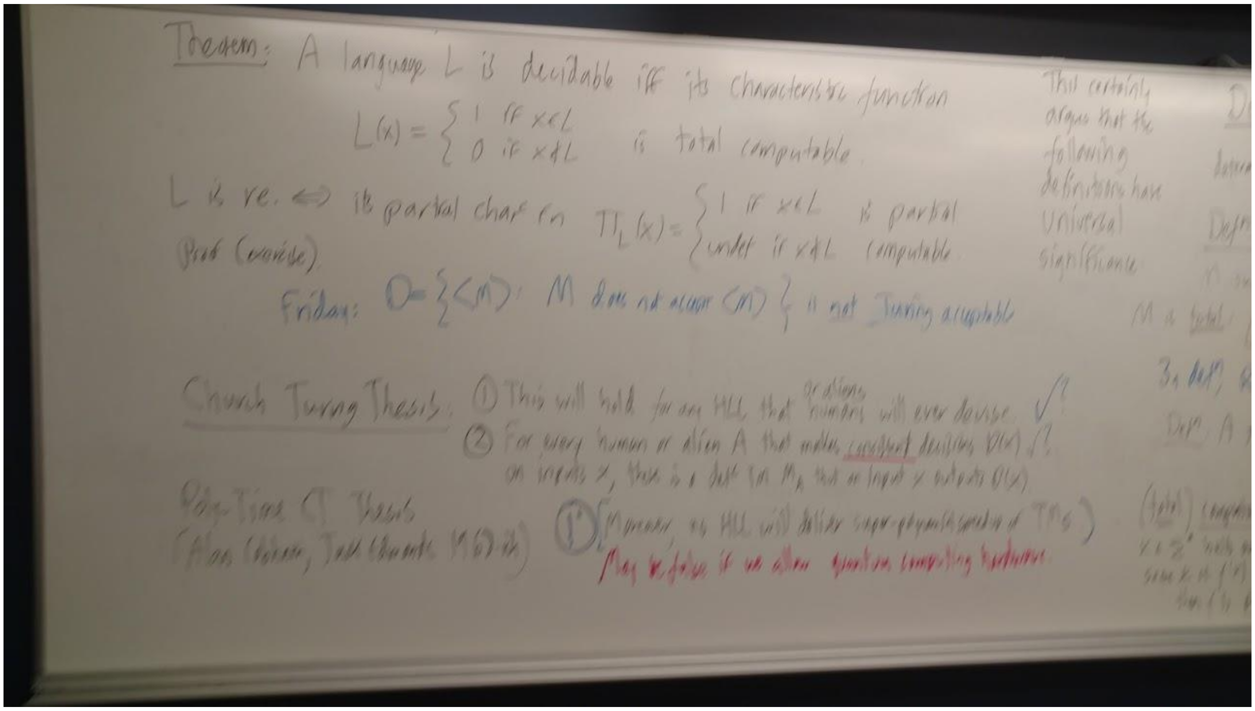
Missing part of definition at top is “Turing acceptable” as a synonym for the other three terms.


Theorem: A language L is decidable iff its characteristic function
L(x) = { 1 if x∈L / 0 if x∉L is total computable
L is r.e. ⟺ its partial char fn Π_L(x) = { 1 if x∈L / undef if x∉L is partial computable
Proof (exercise)
Friday: D = {⟨M⟩ : M does not accept ⟨M⟩} is not Turing acceptable
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② For every human or alien A that makes consistent decisions D(x)
on inputs x, there is a det TM M_A that on input x outputs D(x)
Poly-Time CT Thesis
May be false if we allow quantum computing hardware