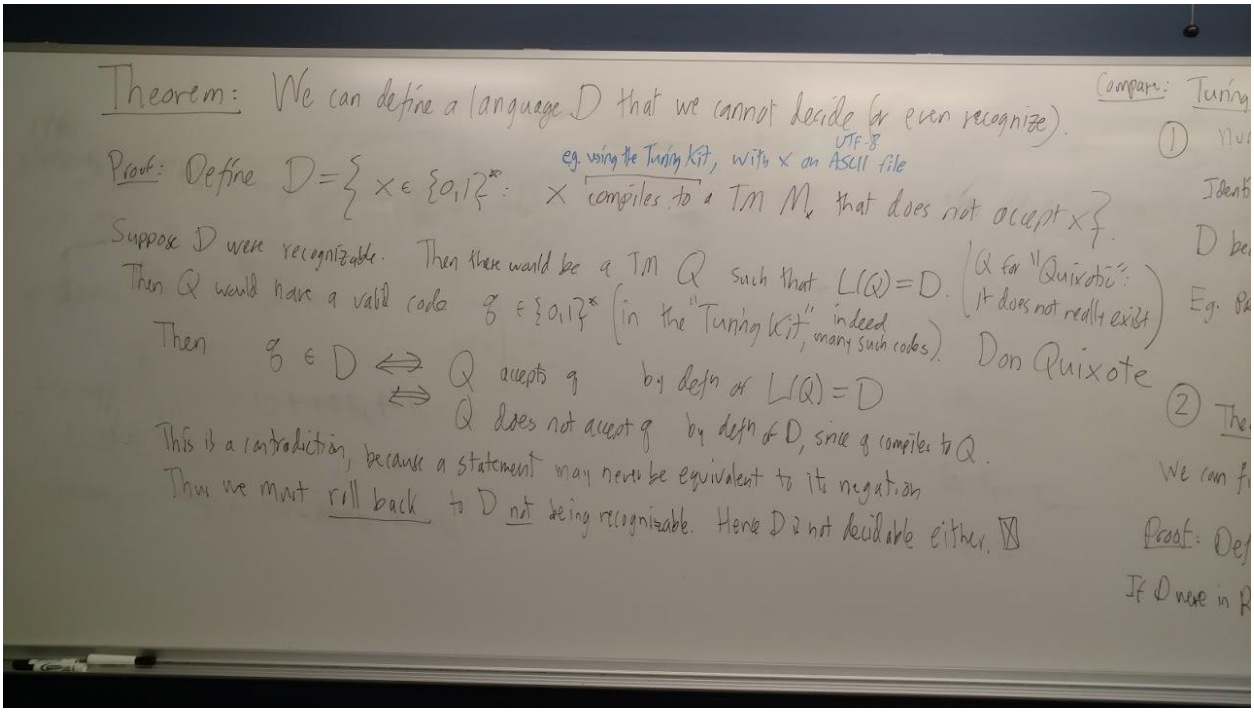
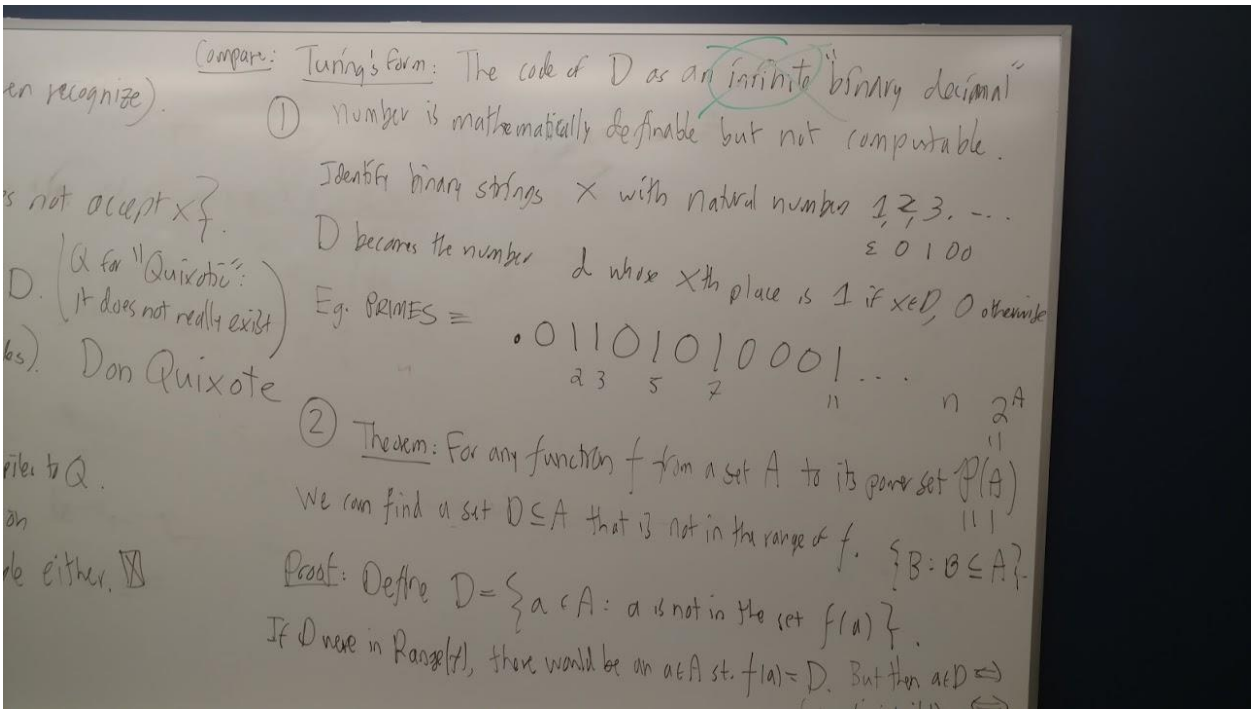
CSE596 Lecture Friday Sept. 27, 2019





Missing text at bottom right says “you finish the proof” (as a self-study exercise).

Theorem: A_{TM} is (recognizable but) not decidable.

Proof: Suppose we had a TM M_U' s.t. $L(M_U') = A_{TM}$ and $\forall \langle M \rangle$ and w, $M_{U'}(\langle M \rangle w)$ always halts.
Then we would get a decider M_D' for $\tilde{D}$ that on any input x would run $M_{U'}(x\,x)$.
But then the TM M_D obtained by interchanging q_{acc} and q_{rej} in M_D' would be a decider for D.

contradicting

Suppose D were recognizable. Then there would be a TM Q such that $L(Q) = D$. (Q for "Quixotic": it does not really exist)
Then Q would have a valid code $q \in \{0,1\}^*$ (in the "Turing Kit", indeed many such codes). Don Quixote

Then $q \in D \iff Q$ accepts q — by defn of $L(Q) = D$
$\iff Q$ does not accept q — by defn of D, since q compiles to Q.

This is a contradiction, because a statement may never be equivalent to its negation.
Thus we must roll back to D not being recognizable. Hence D is not decidable either. ☒ !

However, the com[...]
language $A_{[...]}$
Language of the Acceptance Prob[lem] For TMs
$\tilde{D} = \{$

Further convention:
$\langle M \rangle$ stands for some string x such that x compiles to M and $\langle M \rangle w$ can be parsed as M[...]

Theorem: A_{TM} is (recognizable but) not decidable.

Proof: Suppose we had a TM M_U' s.t. $L(M_U') = A_{TM}$ and $\forall \langle M \rangle$ and w, $M_{U'}(\langle M \rangle w)$ always halts.
Then we would get a decider M_D' for $\tilde{D}$ that on any input x would run $M_{U'}(x\,x)$.
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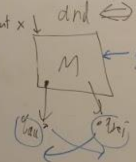
contradicting

Suppose D were recognizable. Then - - - - - - - - - - -

Theorem: A language L is decidable $\iff$ its complement $\tilde{L}$ is decidable
and $\iff$ L and $\tilde{L}$ are both recognizable.

Proof: Given a decider M for L, the machine M' obtained by switching q_{acc} and q_{rej} decides $\tilde{L}$ because M is total.



← Solid Box means every computation path that comes in eventually goes out
here, that means it halts

Proof of ⇐: Monday

(Q for "Quixotic": it does not really exist)
[D]on Quixote

However, the complement
language $A_{TM} = \{$[...]
Language of the Acceptance Problem For TMs.

$\tilde{D} = \{ x : x\#x$[...]

Further convention:
$\langle M \rangle$ stands for some string x such that x compiles to M and $\langle M \rangle w$ can be parsed as M and w.

Here # is a[...]
that $x\#x$[...]
$0 \to 0$[...]
$1 \to 1$[...]
$\# \to$[...]