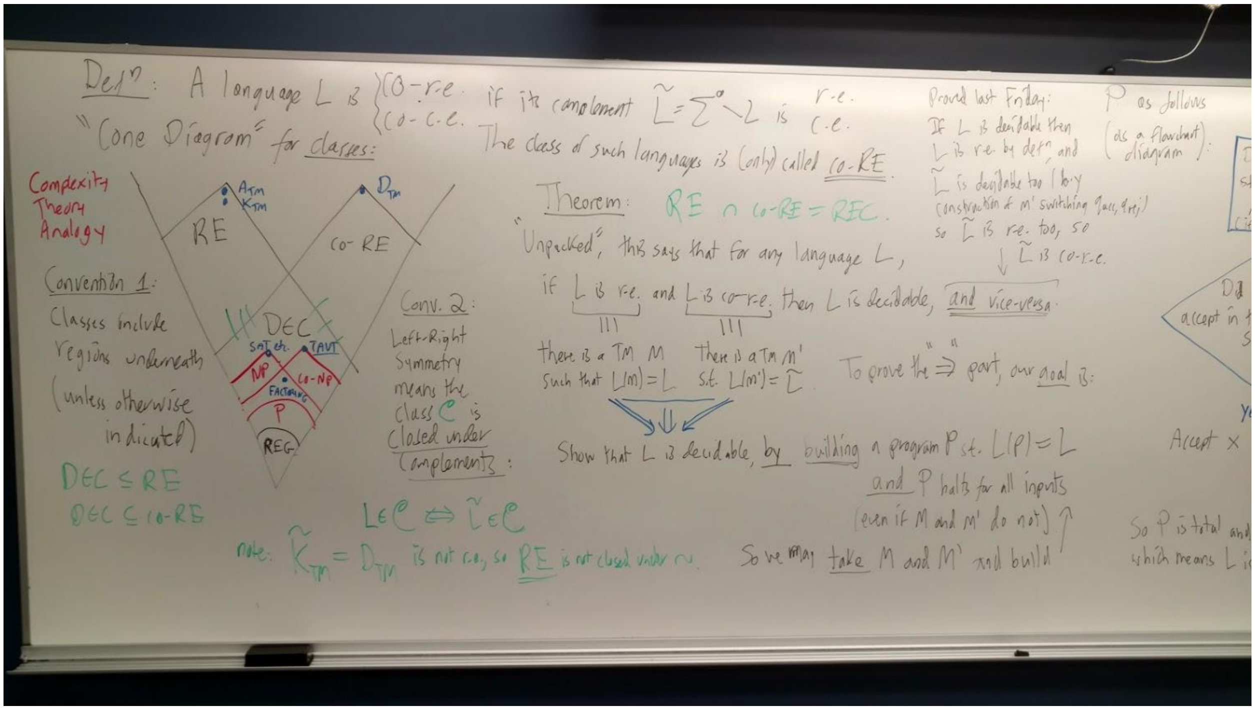
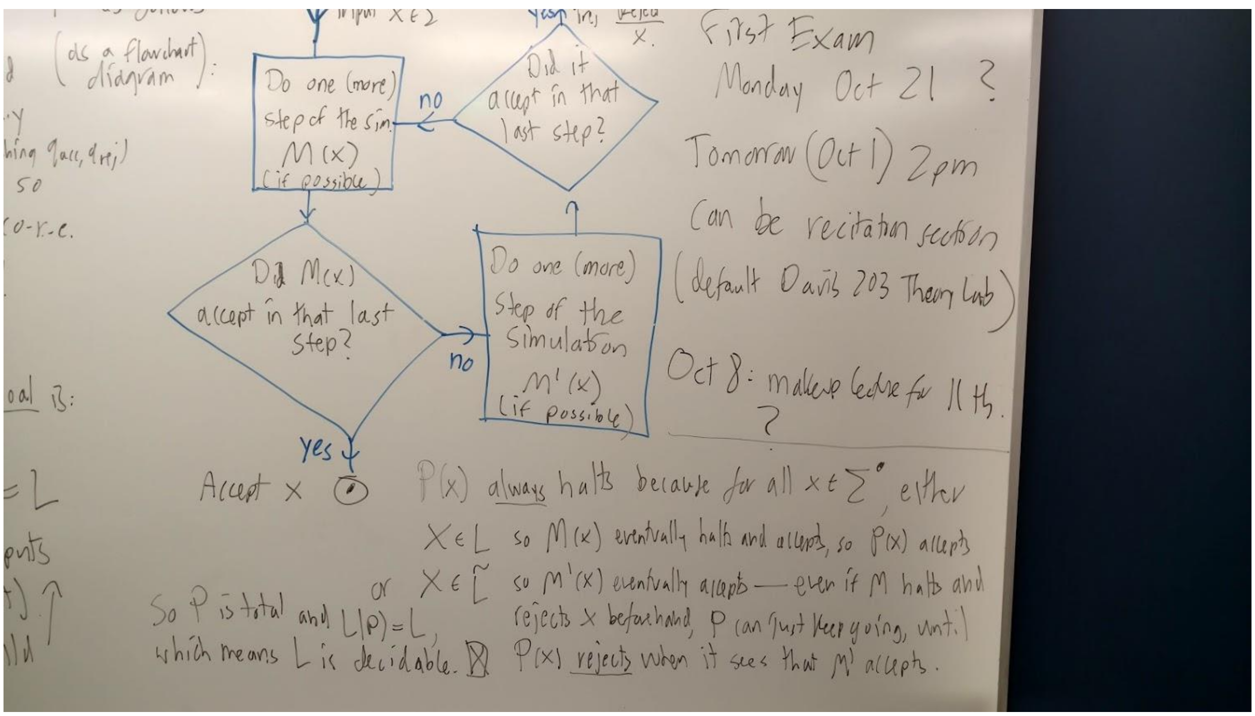
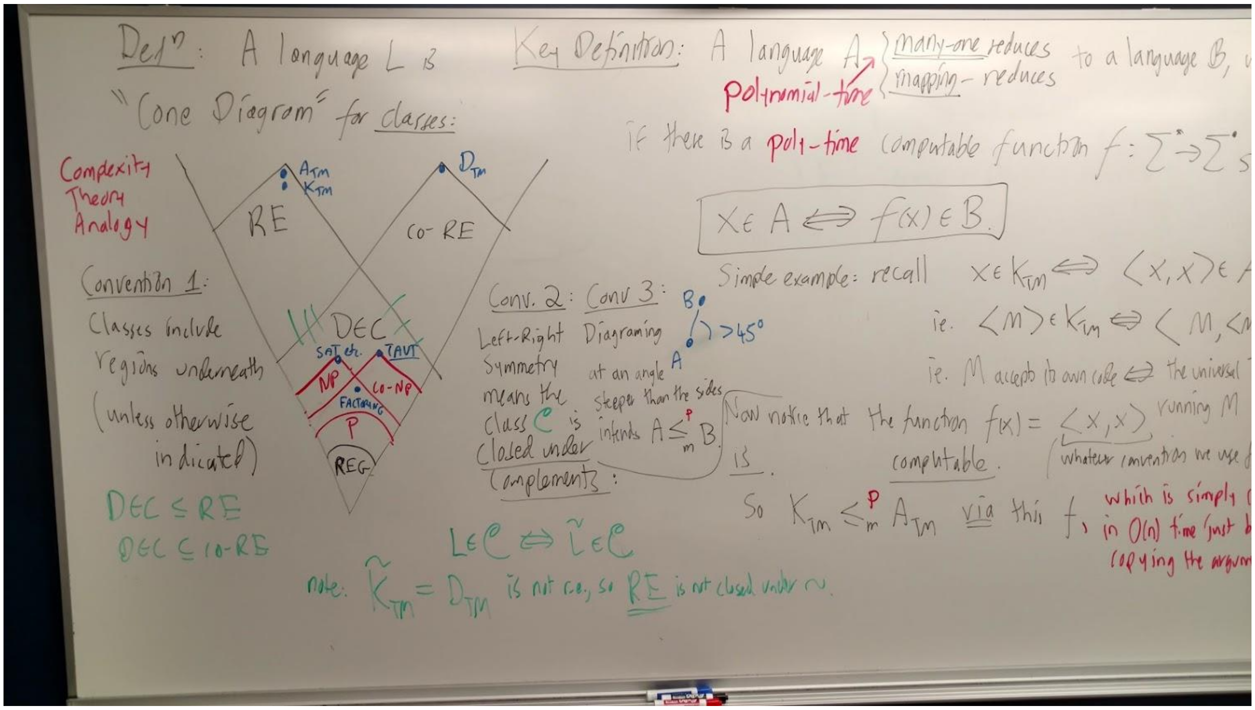
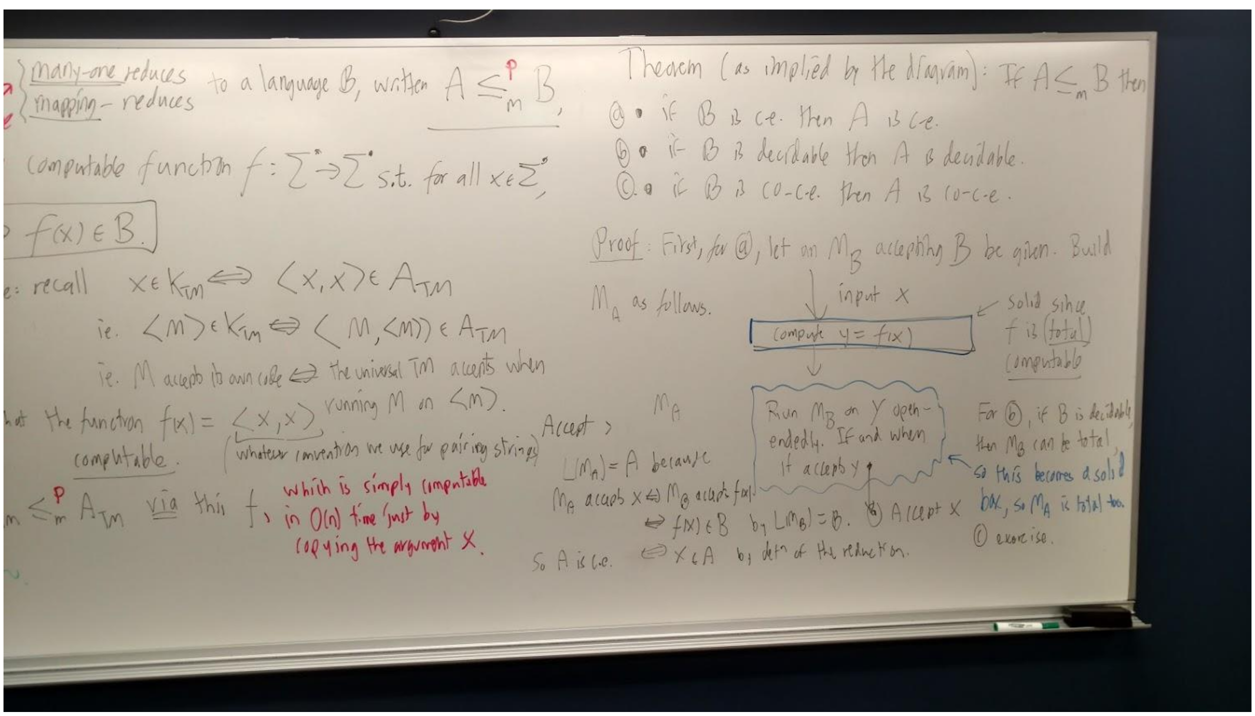
CSE595 Lecture Monday Sept. 30





The lines cut off at the top said “input x \in \Sigma^*” and “yes → Reject x.”


Defn: A language L is
"Cone Diagram" for classes:
Complexity Theory Analogy
A_{TM}
K_{TM}
D_{TM}
RE
co-RE
DEC
SAT etc.
TAUT
NP
co-NP
FACTORING
P
REG
Convention 1: Classes include regions underneath (unless otherwise indicated)
DEC ⊆ RE
DEC ⊆ co-RE
Conv. 2: Left-Right Symmetry means the class $\mathcal{C}$ is closed under complements:
Conv 3: Diagraming at an angle steeper than the sides infers $A \leq_m^p B$.
B
A
>45°
$L \in \mathcal{C} \iff \tilde{L} \in \mathcal{C}$
note: $\tilde{K}_{TM} = D_{TM}$ is not r.e., so RE is not closed under ~.
Key Definition: A language A polynomial-time many-one reduces / mapping-reduces to a language B,
if there is a poly-time computable function $f: \Sigma^* \to \Sigma^*$
$x \in A \iff f(x) \in B.$
Simple example: recall $x \in K_{TM} \iff \langle x, x \rangle \in A$
ie. $\langle M \rangle \in K_{TM} \iff \langle M, \langle M$
ie. M accepts its own code ⟺ the universal
Now notice that the function $f(x) = \langle x, x \rangle$ is computable. (whatever convention we use
running M
So $K_{TM} \leq_m^p A_{TM}$ via this f, which is simply in O(n) time just b copying the argum


many-one reduces / mapping-reduces to a language B, written $A \leq_m^p B$,
computable function $f: \Sigma^* \to \Sigma^*$ s.t. for all $x \in \Sigma^*$,
$f(x) \in B.$
recall $x \in K_{TM} \iff \langle x, x \rangle \in A_{TM}$
ie. $\langle M \rangle \in K_{TM} \iff \langle M, \langle M \rangle \rangle \in A_{TM}$
ie. M accepts its own code ⟺ the universal TM accepts when running M on $\langle M \rangle$.
the function $f(x) = \langle x, x \rangle$ computable. (whatever convention we use for pairing strings)
$\leq_m^p A_{TM}$ via this f, which is simply computable in O(n) time just by copying the argument x.
Theorem (as implied by the diagram): If $A \leq_m B$ then
(a) if B is c.e. then A is c.e.
(b) if B is decidable then A is decidable.
(c) if B is co-c.e. then A is co-c.e.
Proof: First, for (a), let an M_B accepting B be given. Build M_A as follows.
input x
compute y = f(x)
Solid since f is total computable
Run M_B on y open-endedly. If and when it accepts y
Accept x
M_A
Accept
$L(M_A) = A$ because
M_A accepts $x \iff M_B$ accepts $f(x)$
$\iff f(x) \in B$ by $L(M_B) = B$.
$\iff x \in A$ by defn of the reduction.
So A is c.e.
For (b), if B is decidable, then M_B can be total, so this becomes a solid box, so M_A is total too.
(c) exercise.